Neural Control

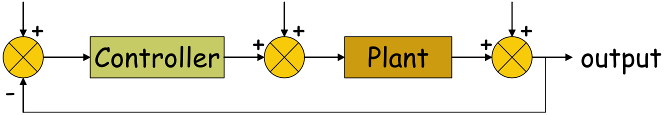
Wen Yu

Departamento de Control Automático
CINVESTAV-IPN
A.P. 14-740, Av.IPN 2508, México D.F., 07360, México

Design goals

- Closed loop follow a prescribed transfer function
  1. Cancellation of non-linearities
  2. Resulting closed-loop transfer function
     - Pole placement
     - Model Reference Control
- Minimize quadratic cost function
  1. Closed loop non-linear
  2. Adaptive
     - Minimum variance
     - Predictive control
     - Optimal control

Feedback control, Linear control

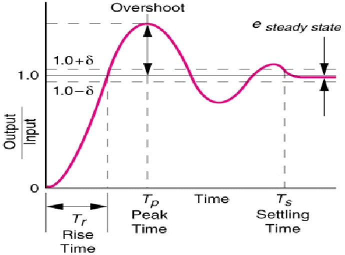


Feedback control

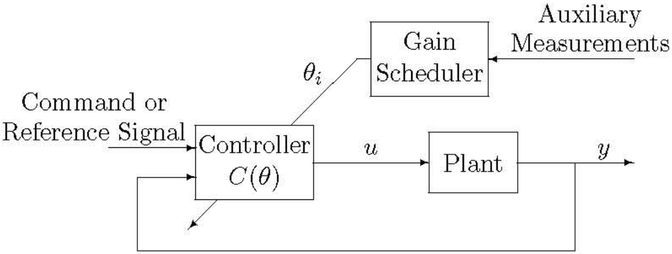
$$u = Kx(t)$$

PID control

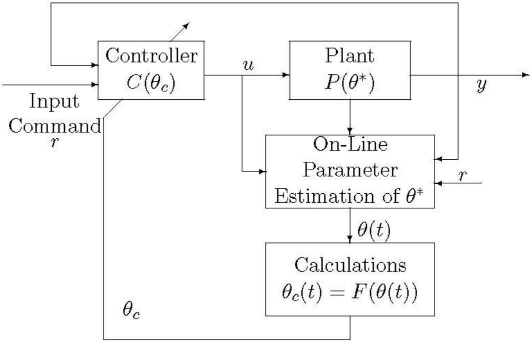
$$u = K_p e(t) + K_i \int e(t)\, dt + K_d \frac{de(t)}{dt}$$



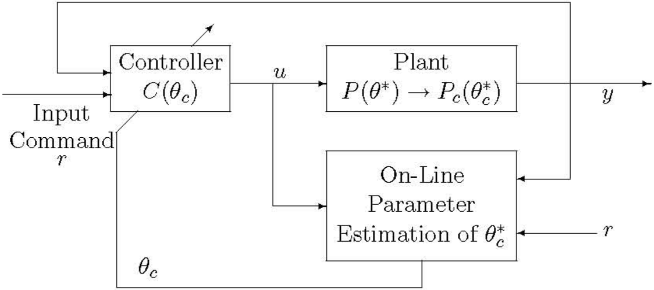
Adaptive control: gain scheduling



Indirect adaptive control



Direct adaptive control



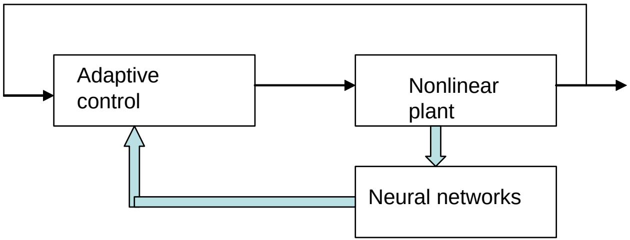
- Linear controllers: small operational ranges
- Hard non-linearities: approximated by linear systems
- Model uncertainties
- Multiple equilibrium: nonlinear systems
- Model-based

Model based/model free control

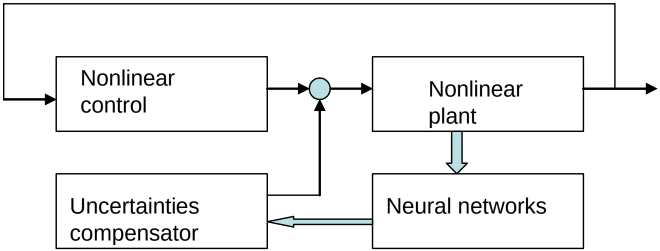
Advance model-based control, feedback linearization, backstepping, direct inverse control, internal model control, MPC

1. Model-based control using data
2. Intelligent methods for controller parameters
3. Intelligent controller as a compensator
4. Knowledge based control

Neual adaptive control



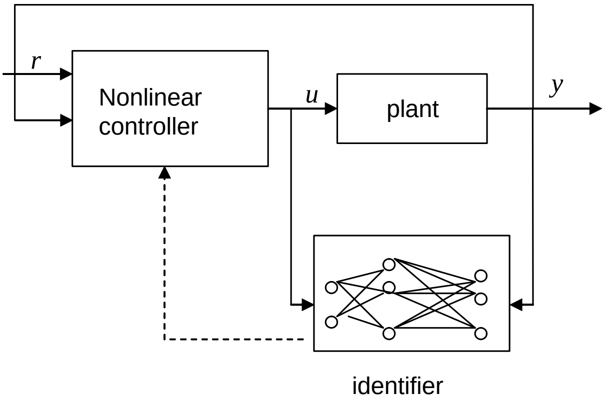
Neural compensation



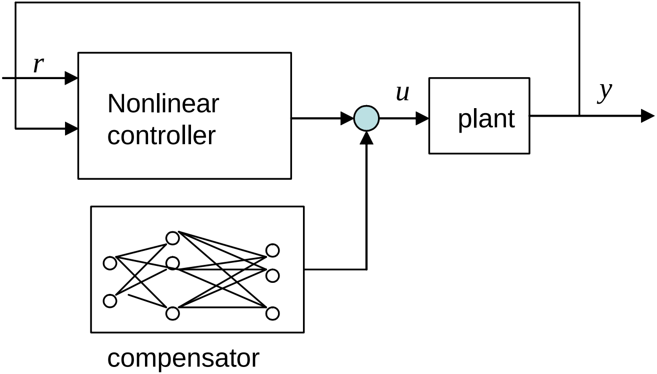
Neural Control

- Before 90's, off-line NN Training
- After 90's, combining adaptive control, and NN parametrization, on-line adaptive NN control is investigated.
  1. neural controller
  2. neural compensator

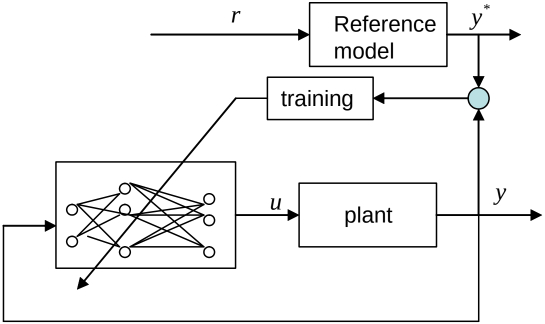
Neural control with identifiier



Neural compensator



Direct inverse control with NN



Direct inverse control

Plant NARMAX model

$$y(k+1) = f\left[y(k), \cdots y(k-n), u(k) \cdots u(k-m)\right]$$

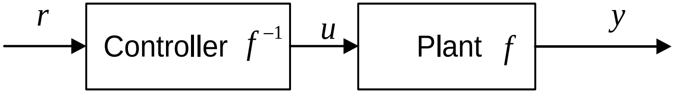
controller

$$u_1(k) = f^{-1}\left[y(k+1), y(k) \cdots y(k-n), u(k) \cdots u(k-m)\right]$$

but $y(k+1)$ is not avaiable, replace $y(k+1)$ with $r(k+1)$

$$u(k) = f^{-1}\left[r(k+1), y(k) \cdots y(k-n), u(k) \cdots u(k-m)\right]$$

If inverse exact, the output is the reference (dead-beat controller)
$y(k+1) = r(k+1)$



Neural direct inverse control

$$u(k) = \hat{f}_{nn}^{-1}\left[r(k+1), y(k)\cdots y(k-n), u(k)\cdots u(k-m)\right]$$

Criterion for training $e(k) = r(k) - y(k)$

$$J(n) = \frac{1}{n}\sum_{k=1}^{n} e^2(k)$$
$$J(k) = J(k-1) + e^2(k)$$

Assume $J(k-1)$ has been minimized,

$$w(k+1) = w(k) - \eta\frac{\partial J}{\partial w}$$

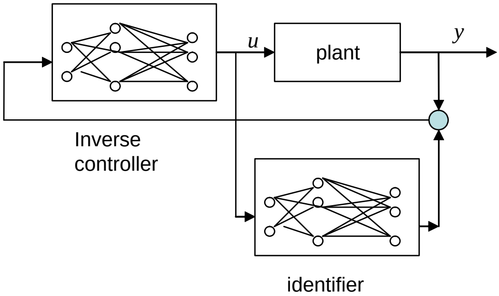
and

$$\frac{\partial J}{\partial w} = \frac{\partial e^2(k)}{\partial w} = -2e(k)\frac{\partial y(k)}{\partial w} = -2e(k)\frac{\partial y(k)}{\partial u(k)}\frac{\partial u(k)}{\partial w(k)}$$

Here $\frac{\partial y(k)}{\partial u(k)}$ is Jacobians of the system, it is a scalar factor to modify the strp size of the algorithm, as long as it has the correct size

$$u(k) = NN(w),$$

Direct inverse control with identifiier



Feedback linearization

If a nonlinear system (standard from, relative degree n) is known,

$$
\begin{aligned}
&\dot{x}_1 = x_2 \\
&\cdots \\
&\dot{x}_n = f\left(x_1 \cdots x_n\right) + g\left(x_1 \cdots x_n\right) u \\
&y = x_1
\end{aligned}
$$

It is a SISO system

$$
\begin{array}{l}
y^{(n)} = f\left(y, \dot{y} \cdots y^{(n-1)}\right) + g\left(y, \dot{y} \cdots y^{(n-1)}\right) u \\
\text{or } x^{(n)} = f\left(x, \dot{x} \cdots x^{(n-1)}\right) + g\left(x, \dot{x} \cdots x^{(n-1)}\right) u
\end{array}
\tag{1}
$$

The control object : find a u such that $y \longrightarrow y_m$. The tracking error is

$$e_c = y_m - y$$

Let

$$e = \left[e_c, \dot{e}_c \cdots e_c^{(n-1)}\right]^T, \quad x = [x_1 \cdots x_n]^T$$

The ideal control is

$$u^* = \frac{1}{g(x)}\left[-f(x) + y_m^{(n)} + K^T e\right]$$

where $K = [k_n \cdots k_1]^T$.

Feedback linearization

The closed-loop system

$$y^{(n)} = y_m^{(n)} + K^T e, \quad e_c^{(n)} + k_1 e_c^{(n-1)} + \cdots + k_n e_c = 0$$

$$\dot{e} = Ae, \quad A = \begin{bmatrix} 0 & 1 & 0 & \cdots 0 \\ & & 1 & \\ & & & \\ -k_0 & \cdots & & -k_{n-1} \end{bmatrix}$$

We select K such that all roots of polynomial

$$s^n + k_1 s^{n-1} + \cdots + k_n$$

are in the left half of complex plane. So

$$\lim_{t\to\infty} e_c(t) = 0$$

Error dynamic

If $f(x)$ is unkonwn, we use a static neural networks to approximate $f(x)$,

$$\hat{f}(x) = W_f \phi_f(x), \quad f(x) = W_f^* \phi_f(x) + \eta_f$$

The feedback linearization with NN is

$$u = \frac{1}{g(x)} \left[-\widehat{f}(x) + y_m^{(n)} + K^T e \right]$$

We define

$$\begin{aligned} \widetilde{f} &= \widehat{f}(x) - f(x) \\ &= W_f \phi_f(x) - W_f^* \phi_f(x) - \eta_f \\ &= \widetilde{W}_f \phi_f(x) - \eta_f, \qquad \widetilde{W}_f = W_f - W_f^* \end{aligned}$$

Stability

The ideal control is

$$u^* = \frac{1}{g(x)}\left[-f(x) + y_m^{(n)} + K^T e\right]$$

The control is substituted to plant $y^{(n)} = f + gu$

$$y^{(n)} = \left[\widehat{f}(x) - \widetilde{f}\right] + g\frac{1}{g}\left[-\widehat{f}(x) + y_m^{(n)} + K^T e\right]$$
$$y^{(n)} = y_m^{(n)} + K^T e - \widetilde{f}$$
$$\dot{e} = Ae - b\widetilde{f} \quad b = [0, \cdots 0, 1]^T$$

Let Lyapunov function

$$V = e^T P e + \frac{1}{k_f}\widetilde{W}_f^T \widetilde{W}_f$$
$$\dot{V} = e^T \left(A^T P + PA\right) e - 2e^T Pb\left(\widetilde{W}_f \phi_f(x) - \eta_f\right) + 2\frac{1}{k_f} tr\left[\widetilde{W}_f^T \dot{\widetilde{W}}_f\right]$$
$$2e^T Pb\eta_f \le e^T Pb\Lambda^{-1} b^T Pe + \overline{\eta}_f$$

Stability

If the larning algorithm is

$$\dot{\widetilde{W}}_f = \dot{W}_f = k_f e^T P b \phi_f$$

and

$$A^T P + PA + Pb\Lambda^{-1} b^T P = -Q$$

then

$$\dot{V} \leq -e^T Q e + \overline{\eta}_f$$

Dead zone updating law

$$\dot{W}_f = \begin{cases} k_f e^T P b \phi_f & \|e\|_Q^2 \geq \overline{\eta}_f \\ 0 & \|e\|_Q^2 < \overline{\eta}_f \end{cases}$$

If $\|e\|_Q^2 \geq \overline{\eta}_f$, $\dot{V} \leq 0$, V is bounded. If $\|e\|_Q^2 < \overline{\eta}_f$, e is bounded and $\dot{W}_f$ is stopped , it is also bounded, V is bounded. Finally $\|e\|_Q^2 \rightarrow \overline{\eta}_f$

Multilayer neural netoworks

We use the following MLP to approximate $f(x)$,

$$\hat{f}(x) = W_f \phi_f (V_f x), \quad f(x) = W_f^* \phi_f (V_f^* x) + \eta_f$$

We define

$$\begin{aligned} \widetilde{f} &= \widehat{f}(x) - f(x) \\ &= W_f \phi_f (V_f x) - W_f^* \phi_f (V_f^* x) - \eta_f \end{aligned}$$

Use Taylor serial

$$f(x) = f(x^*) + (x - x^*)\frac{\partial f}{\partial t} + \delta_f$$

So

$$\begin{aligned}\widetilde{f} &= W_f \phi_f (V_f x) - W_f^* \phi_f (V_f^* x) - \eta_f \\ &= W_f \phi_f (V_f x) - W_f \phi_f (V_f^* x) + W_f \phi_f (V_f^* x) - W_f^* \phi_f (V_f^* x) - \eta_f \\ &= W_f \left(\tilde{V}_f x\right) \dot{\phi}_f + \widetilde{W}_f \phi_f (V_f^* x) - \eta_{1f}\end{aligned}$$

where $\eta_{1f} = \eta_f + \delta_f$

The control is substituted to plant $y^{(n)} = f + gu$

$$\dot{e} = Ae - b\widetilde{f} \quad b = [0, \cdots 0, 1]^T$$

Let Lyapunov function

$$V = e^T Pe + \frac{1}{k_f} tr\left[\widetilde{W}_f^T \widetilde{W}_f\right] + \frac{1}{k_f} tr\left[\widetilde{V}_f^T \widetilde{V}_f\right]$$

$$\dot{V} = e^T \left(A^T P + PA\right) e - 2e^T Pb\left(W_f \left(\tilde{V}_f x\right) \dot{\phi}_f + \widetilde{W}_f \phi_f \left(V_f^* x\right) - \eta_{1f}\right)$$

$$+2\frac{1}{k_f}\left[\widetilde{W}_f^T \dot{\widetilde{W}}_f\right] + 2\frac{1}{k_f}\left[\widetilde{V}_f^T \dot{\widetilde{V}}_f\right]$$

$$2e^T Pb\eta_{1f} \leq e^T Pb\Lambda^{-1} b^T Pe + \overline{\eta}_{1f}$$

Stability of MLP

If the larning algorithm is

$$\dot{W}_f = k_f e^T P b \phi_f \left(V_f^* x \right)$$
$$\dot{V}_f = k_f e^T P b \dot{\phi}_f W_f x$$

and

$$A^T P + PA + Pb\Lambda^{-1} b^T P = -Q$$

then

$$\dot{V} \leq -e^T Q e + \overline{\eta}_{1f}$$

Dead zone updating law

$$s = \begin{cases} k_f e^T P b \phi_f & \|e\|_Q^2 \geq \overline{\eta}_{1f} \\ 0 & \|e\|_Q^2 < \overline{\eta}_{1f} \end{cases}$$

If $\|e\|_Q^2 \geq \overline{\eta}_f$, $\dot{V} \leq 0$, V is bounded. If $\|e\|_Q^2 < \overline{\eta}_f$, e is bounded and $\dot{W}_f$ is stopped , it is also bounded, V is bounded. Finally $\|e\|_Q^2 \to \overline{\eta}_{1f}$.
$\phi_f \left(V_f^* x \right) \to \phi_f \left(V_f^0 x \right)$

g is unkown

If both f and g are unkonwn, we use two neural networks to approximate them,

$$\hat{f}(x) = W_f \phi_f(x), \quad f(x) = W_f^* \phi_f(x) + \eta_f$$
$$\hat{g}(x) = W_g \phi_g(x), \quad g(x) = W_g^* \phi_g(x) + \eta_g$$

The feedback linearization with NN is

$$u = \frac{1}{\hat{g}(x)} \left[-\widehat{f}(x) + y_m^{(n)} + K^T e \right]$$

Error dynamic

We define

$$\begin{aligned}\widetilde{g} &= \widehat{g}\left(x\right) - g\left(x\right) \\ &= W_g \phi_g\left(x\right) - W_g^* \phi_g\left(x\right) - \eta_g \\ &= \widetilde{W}_g \phi_g\left(x\right) - \eta_g\end{aligned}$$

and

$$\begin{aligned}\widetilde{f} &= \widehat{f}\left(x\right) - f\left(x\right) \\ &= W_f \phi_f\left(x\right) - W_f^* \phi_f\left(x\right) - \eta_f \\ &= \widetilde{W}_f \phi_f\left(x\right) - \eta_f\end{aligned}$$

Stability

The control $u = \frac{1}{\hat{g}}\left[-\widehat{f} + y_m^{(n)} + K^T e\right]$ is substituted to plant $y^{(n)} = f + gu$

$$\begin{aligned} y^{(n)} &= \left[\widehat{f} - \widetilde{f}\right] + \left(\widehat{g} - \widetilde{g}\right)\frac{1}{\hat{g}}\left[-\widehat{f} + y_m^{(n)} + K^T e\right] \\ &= y_m^{(n)} + K^T e - \widetilde{f} - \widetilde{g}\frac{1}{\hat{g}}\left[-\widehat{f} + y_m^{(n)} + K^T e\right] \\ &= y_m^{(n)} + K^T e - \widetilde{f} - \widetilde{g}u \\ \dot{e} &= Ae - b\left(\widetilde{f} + \widetilde{g}u\right) \quad b = \left[0, \cdots 0, 1\right]^T \end{aligned}$$

Let Lyapunov function

$$V = e^T Pe + \frac{1}{k_f} \widetilde{W}_f^T \widetilde{W}_f + \frac{1}{k_g} \widetilde{W}_g^T \widetilde{W}_g$$

$$\dot{V} = e^T \left(A^T P + PA\right) e - 2e^T Pb \left(\widetilde{W}_f \phi_f (x) - \eta_f\right) + 2\frac{1}{k_f} tr \left[\widetilde{W}_f^T \dot{\widetilde{W}}_f\right]$$

$$-2e^T Pb \left[\widetilde{W}_g \phi_g (x) - \eta_g\right] u + 2\frac{1}{k_g} tr \left[\widetilde{W}_g^T \dot{\widetilde{W}}_g\right]$$

and

$$2e^T Pb\eta_f \leq e^T Pb\Lambda_f^{-1} b^T Pe + \overline{\eta}_f$$
$$2e^T Pb\eta_g \leq e^T Pb\Lambda_g^{-1} b^T Pe + \overline{\eta}_g$$

Stability

If the larning algorithm is

$$\dot{W}_f = k_f e^T P b \phi_f$$
$$\dot{W}_g = k_g e^T P b \phi_g u$$

and

$$A^T P + PA + P\left(b\Lambda_f^{-1} b^T + b\Lambda_g^{-1} b^T\right) P = -Q$$

then

$$\dot{V} \leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g$$

g is different from zero

We need

$$\hat{g}(x) = W_g \phi_g(x) \neq 0$$

$\phi_g(x)$ can be made such that

$$\left|\phi_g(x)\right| \geq a > 0$$

But how to assure W_g in

$$\dot{W}_g = k_g e^T P b \phi_g u$$

We use projection for W_g such that

$$\|W_g\| \geq b_0$$

Projection

The projection technique

$$\dot{W}_g = \begin{cases} k_g e^T P b \phi_g u & \text{if } \|W_g\| > b_0 \\ & \text{or } \|W_g\| = b_0 \text{ and } \left(e^T P b \phi_g u\right)(W_g) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The projection condition is

- if $W_g \geq 0$, $k_g e^T P b \phi_g u > 0$, so $\|W_g\| \uparrow$
- if $W_g < 0$, $k_g e^T P b \phi_g u < 0$, so $\|W_g\| \uparrow$

It assues $|W_g| \geq b > 0$.

Stability of projection

$$\dot{W}_f = k_f e^T P b \phi_f$$
$$\dot{W}_g = s k_g e^T P b \phi_g u$$

Lyapunov

$$V = e^T P e + \frac{1}{k_f} \widetilde{W}_f^T \widetilde{W}_f + \frac{1}{s k_g} \widetilde{W}_g^T \widetilde{W}_g$$

$$s = \begin{cases} 1 & \text{if } \|W_g\| > b_0 \text{ or } \|W_g\| = b_0 \text{ and } \left(e^T P b \phi_g u\right)(W_g) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Because

$$\dot{V} \leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g + \left[\frac{1}{s k_g} \dot{\widetilde{W}}_g - e^T P b \phi_g (x) u \right] \widetilde{W}_g$$

Stability of projection

If $|W_g| > b_0$ or $|W_g| = b_0$ and $\left(e^T P b \phi_g u\right)(W_g) \geq 0$, $s = 1$

$$\dot{V} \leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g$$

if $|W_g| = b_0$, it is a constant, $\dot{\widetilde{W}}_g = 0$, $\dot{\widetilde{W}}_g \widetilde{W}_g = 0$,
If $W_g < 0$, $\left(e^T P b \phi_g u\right) > 0$,
$2e^T P b \widetilde{W}_g \phi_g(x) u = tr\left\{\left(-b_0 - W^0\right)\left[e^T P b \phi_g u\right]\right\} < 0.$
If $W_g \geq 0$, $\left(e^T P b \phi_g u\right) < 0$,
$2e^T P b \widetilde{W}_g \phi_g(x) u = tr\left\{\left(b_0 - W^0\right)\left[e^T P b \phi_g u\right]\right\} < 0$

$$\begin{aligned}\dot{V} &\leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g + 2e^T P b \widetilde{W}_g \phi_g(x) u \\ &\leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g\end{aligned}$$

In both cases

$$\dot{V} \leq -e^T Q e + \overline{\eta}_f + \overline{\eta}_g$$

Stability of projection and dead-zone

Is it $\|e\|_Q^2 \to \left(\overline{\eta}_f + \overline{\eta}_g\right)$?

$$\dot{W}_f = s_f k_f e^T P b \phi_f$$
$$\dot{W}_g = s_g k_g e^T P b \phi_g u$$

$$s_f = \begin{cases} 1 & \text{if } \|e\|_Q^2 \geq \left(\overline{\eta}_f + \overline{\eta}_g\right) \\ 0 & \text{otherwise} \end{cases}$$

$$s_g = \begin{cases} & \text{if } \|W_g\| > b \text{ or } \|W_g\| = b \\ 1 & \text{and } \left(e^T P b \phi_g u\right)(W_g) \geq 0 \\ & \text{and } \|e\|_Q^2 \geq \left(\overline{\eta}_f + \overline{\eta}_g\right) \\ 0 & \text{otherwise} \end{cases}$$

$\|e\|_Q^2 \to \left(\overline{\eta}_f + \overline{\eta}_g\right)$ and all signals are bounded

Projection modification : parameter bounded

$$\dot{W}_f = s_f k_f e^T P b \phi_f$$

$$\dot{W}_f = \begin{cases} k_f e^T P b \phi_f & \text{if } \|W_f\| < M_f \text{ or } \|W_f\| = M_f \\ & \text{and } \left(e^T P b \phi_f\right) W_f < 0 \\ \Pr\left(k_f e^T P b \phi_f\right) & \text{otherwise} \end{cases}$$

where

$$\Pr\left(k_f e^T P b \phi_f\right) = k_f e^T P b \phi_f - k_f e^T P b \phi_f \frac{W_f}{\|W_f\|^2}$$

Projection modification: parameter bounded -proof

Lyapunov function

$$V_f = tr\left(W_f^T W_f\right)$$
$$\dot{V}_f = 2tr\left(W_f^T \dot{W}_f\right)$$

If $\|W_f\| = M_f$ and $\left(e^T P b \phi_f\right) W_f \leq 0$

$$\dot{V}_f = 2tr\left(W_f^T\left(k_f e^T P b \phi_f\right)\right) \leq 0, \qquad \|W_f\| \downarrow$$

If $\|W_f\| = M_f$ and $\left(e^T P b \phi_f\right) W_f > 0$

$$\dot{V}_f = 2tr\left(W_f^T\left(k_f e^T P b \phi_f\right) - k_f e^T P b \phi_f \frac{W_f^T W_f}{\|W_f\|^2}\right) = 0$$

$\|W_f\|$ is constant