PID Control with Intelligent Compensation

Wen Yu

https://www.ctrl.cinvestav.mx/~yuw/
yuw@ctrl.cinvestav.mx

Introduction

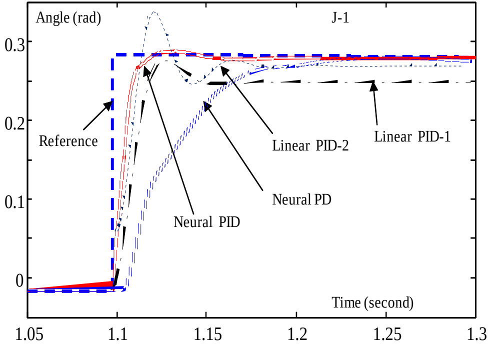
PID control (Proportional-Integral-Derivative control)
The error between a desired set-point and a measured process variable

$$e = y^* - y$$

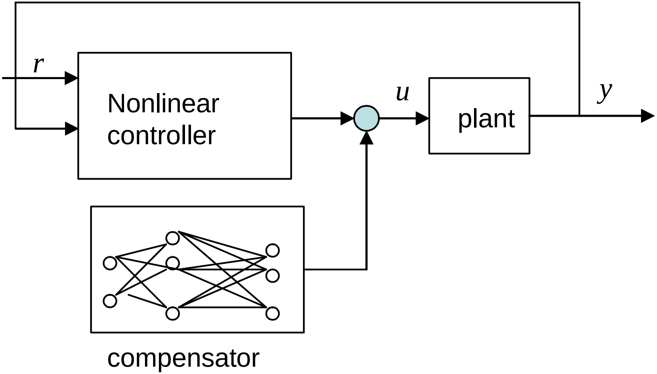
The control signal to the process to minimize the error is

$$u = K_p e + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}$$

- Proportional (P): If the error is large, the controller output will also be large.
- Integral (I): It helps to eliminate steady-state errors.
- Derivative (D): It helps to anticipate future errors and reduce overshoot.



Neural compensator

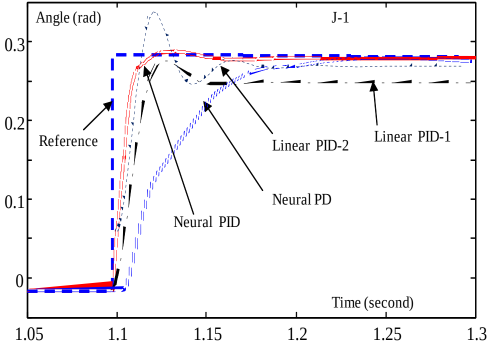


Motivations

PID (model-free) + Intelligent control (model-free)

Performance + Stability

- **Improved Performance**
- Enhanced Robustness:
- Increased Flexibility



Stability of PD

The dynamics of a robot

$$M\left(q\right)\ddot{q}+C\left(q,\dot{q}\right)\dot{q}+g\left(q\right)+f\left(\dot{q}\right)=\tau$$

The classical industrial PD law is

$$\tau=K_p(q^d-q)+K_d(\dot{q}^d-\dot{q})$$

Theorem

The tracking error

$$e=q^*-q$$

is bounded

$$\left\|e\right\|^2<\infty$$

when

$$K_p>0,\qquad K_d>0$$

Proof of PD cotnrol

The dynamics of a serial $n-$link rigid robot manipulator

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau$$

Property 1. The inertia matrix is symmetric and positive definite , *i.e.*

$$m_1 \|x\|^2 \leq x^T M(x_1) x \leq m_2 \|x\|^2 ; \forall x \in R^n$$

Property 2. The centripetal and Coriolis matrix is skew-symmetric,

$$x^T \left[\dot{M}(q) - 2C(q,\dot{q}) \right] x = 0$$

Proof of PD cotnrol

The tracking error is defined as

$$e = q^d - q, \quad \dot{e} = \dot{q}^d - \dot{q}$$

Normal PD control is

$$\tau = K_p e + K_d \dot{e}$$

where K_p and K_d are positive definite, symmetric and constant matrices

Proof of PD cotnrol: Regulation case

In regulation case

$$\dot{q}^d = 0, \quad \dot{e} = \dot{q}^d - \dot{q} = -\dot{q}, \quad \ddot{e} = -\ddot{q}$$

When G and F are known,

$$\tau = K_p e + K_d \dot{e} + G + F$$

The proposed Lyapunov function is

$$V = \frac{1}{2}\dot{e}^T M \dot{e} + \frac{1}{2} e^T K_p e$$

The derivative of V is

$$\dot{V} = \dot{e}^T M \ddot{e} + \frac{1}{2}\dot{e}^T \dot{M} \dot{e} + \dot{e}^T K_p e$$

Proof of PD cotnrol: Regulation case

Since $M\ddot{e} = -M\ddot{q}$ and $C\dot{q} + G + F - \tau = -M\ddot{q}$,

$$M\ddot{e} = C\dot{q} + G + F - \tau$$

and

$$\begin{aligned} \dot{e}^T M\ddot{e} &= \dot{e}^T \left(C\dot{q} + G + F - \tau\right) \\ &= -\dot{e}^T C\dot{e} + \dot{e}^T \left(G + F - \tau\right) \end{aligned}$$

So

$$\begin{aligned} \dot{V} &= -\dot{e}^T C\dot{e} + \dot{e}^T \left(G + F - \tau\right) + \tfrac{1}{2}\dot{e}^T \dot{M}\dot{e} + \dot{e}^T K_p e \\ &= \tfrac{1}{2}\dot{e}^T \left[\dot{M} - 2C\right]\dot{e} + \dot{e}^T \left(G + F - \tau\right) + \dot{e}^T K_p e \end{aligned}$$

From *Property 2*, $x^T \left[\dot{M} - 2C\right] x = 0$ and $\tau = K_p e + K_d\dot{e} + G + F$,

$$\dot{V} = -\dot{e}^T \left(K_p e + K_d\dot{e}\right) + \dot{e}^T K_p e = -\dot{e}^T K_d\dot{e} \leq 0$$

LaSalle

Proof of PD cotnrol: Tracking case

$\dot{q}^d \neq 0$, so $\dot{e} \neq -\dot{q}$, $\ddot{e} \neq -\ddot{q}$, the tracking error as:

$$e = q^d - q, \quad \dot{e} = \dot{q}^d - \dot{q}$$

Define

$$r = \dot{e} + \Lambda e, \quad \Lambda = \Lambda^T > 0$$

Lyapunov

$$V = \frac{1}{2} r^T M r$$

The derivative of V is

$$\dot{V} = r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r$$

Using $C\dot{q} + G + F - \tau = -M\ddot{q}$, $-Cr + C\dot{q}^d + C\Lambda e = C\dot{q}$

$$\begin{aligned} M\dot{r} &= M\ddot{e} + M\Lambda\dot{e} = M\ddot{q}^d - M\ddot{q} + M\Lambda\dot{e} \\ &= M\ddot{q}^d + C\dot{q} + G + F - \tau + M\Lambda\dot{e} \\ &= -\tau - Cr + \left[M\left(\Lambda\dot{e} + \ddot{q}^d\right) + C\left(\Lambda e + \dot{q}^d\right) + G + F\right] \end{aligned}$$

Proof of PD cotnrol: Tracking case

If the control is

$$\tau = Kr + f + G + F$$

where $f = M\left(\Lambda\dot{e} + \ddot{q}^d\right) + C\left(\Lambda e + \dot{q}^d\right)$, then

$$M\dot{r} = -Kr - Cr$$

$$\begin{aligned}\dot{V} &= r^T\left(-Kr - Cr\right) + \tfrac{1}{2}r^T\dot{M}r \\ &= -r^TKr - r^TCr + \tfrac{1}{2}r^T\dot{M}r\end{aligned}$$

From *Property 2*, $r^T\left[\dot{M} - 2C\right]r = 0$

$$\dot{V} = -r^TKr \leq 0$$

But we need kto know

$$f = M\left(\Lambda\dot{e} + \ddot{q}^d\right) + C\left(\Lambda e + \dot{q}^d\right), \rightarrow M, C, G, F$$

Stability of PD

Theorem

For regulation case $\frac{d}{dt}q^* = 0$, $g(q)$ *and* $f(\dot{q})$ *are known, and*

$$\tau = PD + g(q) + f(\dot{q})$$

then

$$\lim_{t\to\infty} \|e\|^2 = 0$$

if

$$K_p > 0, \qquad K_d > 0$$

Stability of PD with gravity compensation

Theorem

When only $g(q)$ *is known,* $f\left(\dot{q}\right)$ *is unknown, if the control is*

$$\tau = PD + g(q)$$

and

$$K_p > 0, \qquad K_d > K_1 > 0$$

then

$$\|e\|^2 \leq \bar{d}$$

$\bar{d}$ *is upper bound of the friction* $f(\dot{q})$.

PD with neural network compensation

When $g(q)$ and $f\left(\dot{q}\right)$ are known, the control is

$$\tau = PD + g(q) + f(\dot{q})$$

and

$$\lim_{t\to\infty} \|e\|^2 = 0$$

When $g(q)$ and $f\left(\dot{q}\right)$ are un known, PD with neural network compensation is

$$u = PD + NN$$

where $NN = W_t\sigma(x)$, and

$$g(q) + f\left(\dot{q}\right) = W_t\sigma(x) + \varepsilon$$

PD with neural network compensation

Theorem

If the PD control with neual compensator is

$$\tau = PD + W_t \sigma(x), \quad K_p > 0, K_d > K_1 > 0$$

where the weight W_t *is updated as*

$$\frac{d}{dt} W_t = K_w \sigma(x) e^T$$

then the tracking error converges to $\bar{d}$

$$\|e\|^2 \leq \bar{d}$$

here $\bar{d}$ *is the upper bound of the nerual approximation error* ε*, i.e.,*

$$g(q) + f(\dot{q}) = W_t \sigma(x) + \varepsilon$$

Proof of PD cotnrol with neural compensation

If the model is unknown, the PD control is

$$\tau = Kr$$

but

$$\begin{aligned} M\dot{r} &= -\tau - Cr + \left[M\left(\Lambda\dot{e} + \ddot{q}^d\right) + C\left(\Lambda e + \dot{q}^d\right) + G + F\right] \\ &= -Kr - Cr + f + G + F \end{aligned}$$

Then

$$\begin{aligned} \dot{V} &= -r^T Kr - r^T Cr + \tfrac{1}{2}\dot{e}^T \dot{M}\dot{e} + r^T\left(f + G + F\right) \\ &= -r^T Kr + r^T\left(f + G + F\right) \end{aligned}$$

Proof of PD cotnrol with neural compensation

Estimation of $r^T(f+G+F)$

$$r^T(f+G+F) \leq r^T\Lambda r + (f+G+F)^T\Lambda^{-1}(f+G+F) \leq r^T\Lambda r + \bar{d}$$

where $\bar{d}$ is upper bound of $(f+G+F)^T\Lambda^{-1}(f+G+F)$

$$\dot{V} \leq -r^T(K-\Lambda)r + \bar{d}$$

when $K > \Lambda$

$$\|r\|^2_{(K-\Lambda)} \to \bar{d}$$

Proof of PD cotnrol with neural compensation

The PD control with neural compensation

$$\tau = Kr + W\sigma(x)$$

here

$$W\sigma(x) \approx G + F - f$$

where $x = \left[q^T, \dot{q}^T, e^T, \dot{e}^T, q^d, \dot{q}^d, \ddot{q}^d\right]^T$, also

$$G + F - f = W^*\sigma(x) + \eta$$

where η is bounded modeling error, $\|\eta\|^2 < \bar{\eta}$

Proof of PD cotnrol with neural compensation

The proposed Lyapunov function is

$$V = \frac{1}{2} r^T M r + \frac{1}{2} tr\left(\tilde{W}^T K_w^{-1} \tilde{W}\right)$$

where $\tilde{W} = W^* - \tilde{W} -$. The derivative of it is

$$\begin{aligned} \dot{V} &= r^T M \dot{r} + \tfrac{1}{2} r^T \dot{M} r + tr\left[\tilde{W}^T K_w^{-1} \left(\tfrac{d}{dt}\tilde{W}\right)\right] \\ &= -r^T K r + r^T \left(f + G + F\right) + tr\left[\tilde{W}^T K_w^{-1} \left(\tfrac{d}{dt}\tilde{W}\right)\right] \end{aligned}$$

Using neural compensator $\tau = Kr + W\sigma(x)$,

$$\begin{aligned} M\dot{r} &= -Kr - Cr + f + G + F \\ &= -Kr - W\sigma(x) + W^*\sigma\left(x\right) + \eta \\ &= -Kr - Cr + \tilde{W}\sigma\left(x\right) + \eta \end{aligned}$$

Now

$$\dot{V} = -r^T K r + r^T \left(\tilde{W}\sigma\left(x\right) + \eta\right) + tr\left[\tilde{W}^T K_w^{-1} \left(\frac{d}{dt}\tilde{W}\right)\right]$$

Proof of PD cotnrol with neural compensation

The modeling error is estimated as

$$r^T\eta \le r^T\Lambda r + \eta^T\Lambda^{-1}\eta \le r^T\Lambda r + \overline{\eta}$$

$$\dot{V} \le -r^T\left(K-\Lambda\right)r + \overline{\eta} + tr\left[\tilde{W}^T\left(K_w^{-1}\left(\frac{d}{dt}\tilde{W}\right) + \sigma(x)r^T\right)\right]$$

We let

$$K_w^{-1}\left(\frac{d}{dt}\tilde{W}\right) + \sigma(x)r^T = 0$$

the updating law is

$$\frac{d}{dt}\tilde{W} = \frac{d}{dt}W = -K_w\sigma(x)r^T$$

then

$$\dot{V} \le -r^T\left(K-\Lambda\right)r + \overline{\eta}$$

when $K > \Lambda$

$$\|r\|^2_{(K-\Lambda)} \to \overline{\eta}$$

here $\overline{\eta} \ll \bar{d}$, because $G + F - f = W^*\sigma(x) + \eta$.

PD with MLP compensation

$$g\left(q\right)+f\left(\dot{q}\right)=W_t\sigma(V_t x)+\varepsilon$$

PD with MLP compensation

Theorem

If the PD control with neual compensator is

$$\tau = PD + W_t\sigma(V_t x), \quad K_p > 0, K_d > K_1 > 0$$

where the weight W_t *and* V_t *are updated as*

$$\begin{array}{c} \frac{d}{dt}W_t = -d_t K_w \sigma(V_t x)\overline{x}_2^T - d_t K_w D_\sigma V_v x \overline{x}_2^T \\ \frac{d}{dt}V_t = -d_t K_v \overline{x}_2^T D_\sigma W_t x + d_t l K_v x^T V_t x \Lambda_3 \end{array}$$

$$d_t = \begin{cases} 0 & if \quad \|\overline{x}_2\|_R^2 \leq \frac{\chi^2}{4\lambda_{\min}(\Gamma)} \\ 1 & if \quad \|\overline{x}_2\|_R^2 > \frac{\chi^2}{4\lambda_{\min}(\Gamma)} \end{cases}$$

then the tracking error converges to $\bar{d}$

$$\left\{ \overline{x}_2 \mid \|e\|_R^2 \leq \frac{\bar{d}}{4\lambda_{\min}(\Gamma)} \right\}$$

From PD to PID

$$u = K_p e + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}$$

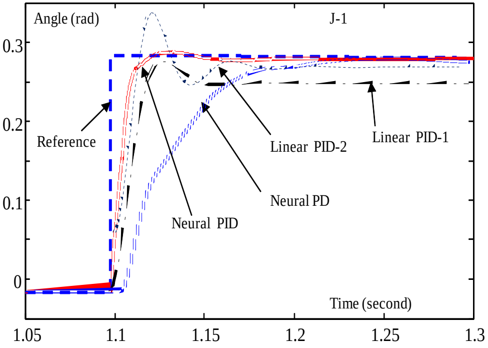


Figure:

PID

PID control law can be expressed

$$\begin{aligned} u &= K_p e - K_d \dot{e} + z \\ \dot{z} &= K_i e \end{aligned}$$

with $e(0) = 0$. The closed-loop system is

$$\frac{d}{dt} \begin{bmatrix} z \\ e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} K_i e \\ -\dot{e} \\ \ddot{q}^d + M^{-1} \left(C\dot{q} + g - K_p \tilde{q} + K_d \dot{q} - z \right) \end{bmatrix}$$

Stability of PID

Theorem

Consider robot dynamic controlled by linear PID controller, the closed loop system is semi-globally asymptotically stable at the equilibrium,

$$\|e\|^2 \leq \frac{\lambda_M(M)}{\alpha K}, \qquad K > 0$$

provided that control gains satisfy

$$\begin{array}{l} \lambda_m(K_p) \geq \frac{3}{2} k_g \\ \lambda_M(K_i) \leq \beta \frac{\lambda_m(K_p)}{\lambda_M(M)} \\ \lambda_m(K_d) \geq \beta + \lambda_M(M) \end{array}$$

PID+NN

Consider robot dynamic

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = u$$

Neural PID control

$$\begin{aligned} \tau &= K_p e + K_d \dot{e} + z + W\sigma(q,\dot{q}) \\ \dot{z} &= K_i e, \quad z(0) = z_0 \end{aligned}$$

Theorem

Consider robot dynamic controlled by the neural PID control, the closed loop system is semiglobally asymptotically stable at the equilibrium $x = \left[\xi - \phi\left(q^d\right), e, \dot{e}\right]^T = 0$

$$\|e\| \leq \frac{\lambda_M\left(M\right)}{K_c}\lambda_M\left(K_i\right)\lambda_m\left(K_p\right) \tag{1}$$

provided that control gains satisfy

$$\lambda_m\left(K_p\right) \geq \frac{3}{2}k_\phi, \lambda_M\left(K_i\right) \leq \beta\frac{\lambda_m\left(K_p\right)}{\lambda_M\left(M\right)}, \lambda_m\left(K_d\right) \geq \beta + \lambda_M\left(M\right) \tag{2}$$

where $\beta = \sqrt{\frac{\lambda_m(M)\lambda_m(K_p)}{3}}$, $k_\phi > 0$, *and the weight of the neural networks is tuned by*

$$\frac{d}{dt}W = -K_w\sigma(q,\dot{q})r^T, \qquad r = \dot{e} + \Lambda e$$

Proof of of PID+NN

$$G + F - f = W^{*}\sigma(x) + \eta\left(x\right)$$

the lower bound of $\int \eta\left(x\right)dx$ is k_{ϕ}
We construct a Lyapunov function as

$$\begin{array}{l} V = V_1 + V_2 + V_3 + V_4 \\ V_1 = \frac{1}{6}e^{T}K_{p}e + e^{T}\eta + \frac{3}{2}\eta^{T}K_{p}^{-1}\eta \\ V_2 = \frac{1}{6}e^{T}K_{p}e + e^{T}z + \frac{\alpha}{2}z^{T}K_{i}^{-1}z \\ V_3 = \frac{1}{6}e^{T}K_{p}e - \Lambda e^{T}M\dot{e} + \frac{1}{2}\dot{e}^{T}M\dot{e} \\ V_4 = \int_0^t \eta dx - k_{\phi} + \frac{\alpha}{2}e^{T}K_{d}e + \frac{1}{2}tr\left(\tilde{W}^{T}K_{w}^{-1}\tilde{W}\right) \end{array} \quad (3)$$

Proof of of PID+NN

We first prove V is a Lyapunov function, $V \geq 0$.

$$V_4 = \int_0^t \eta dx - k_\phi + \frac{\alpha}{2} e^T K_d e + \frac{1}{2} tr\left(\tilde{W}^T K_w^{-1} \tilde{W}\right) \geq 0$$

$$V_1 = \frac{1}{2} \begin{bmatrix} e \\ \eta \end{bmatrix}^T \begin{bmatrix} \frac{1}{3} K_p & I \\ I & 3K_p^{-1} \end{bmatrix} \begin{bmatrix} e \\ \eta \end{bmatrix} \quad (4)$$

Since $K_p \geq 0$, V_1 is a semi positive definite matrix, $V_1 \geq 0$.
When $\alpha \geq \frac{3}{\lambda_m\left(K_i^{-1}\right)\lambda_m(K_p)}$,

$$V_2 \geq \frac{1}{2}\left(\sqrt{\frac{1}{3}\lambda_m\left(K_p\right)}\|e\| - \sqrt{\frac{3}{\lambda_m\left(K_p\right)}}\|z\|\right)^2 \geq 0 \quad (5)$$

Proof of of PID+NN

Because

$$y^T Ax \leq \|y\| \|Ax\| \leq \|y\| \|A\| \|x\| \leq |\lambda_M (A)| \|y\| \|x\| \tag{6}$$

when $\alpha \leq \frac{\sqrt{\frac{1}{3}\lambda_m(M)\lambda_m(K_p)}}{\lambda_M(M)}$,

$$V_3 \geq \frac{1}{2}\left(\sqrt{\lambda_m (M)} \|\dot{e}\| - \sqrt{\frac{1}{3}\lambda_m (K_p)} \|e\|\right)^2 \geq 0 \tag{7}$$

If

$$\sqrt{\frac{1}{3}}\lambda_m \left(K_i^{-1}\right) \lambda_m^{\frac{3}{2}} (K_p) \lambda_m^{\frac{1}{2}} (M) \geq \lambda_M (M) \tag{8}$$

there exists

$$\frac{\sqrt{\frac{1}{3}\lambda_m (M) \lambda_m (K_p)}}{\lambda_M (M)} \geq \Lambda \geq \frac{3}{\lambda_m \left(K_i^{-1}\right) \lambda_m (K_p)} \tag{9}$$

This means if K_p is sufficiently large or K_i is sufficiently small, $V\left(\dot{q}, \tilde{q}, \tilde{\xi}\right)$ is globally positive definite.

Proof of of PID+NN

Using $\frac{d}{dt}\int_0^t \eta = \frac{\partial \int_0^t \eta}{\partial x}\frac{\partial x}{\partial t} = \dot{x}^T\eta$, $\frac{d}{dt}\eta = 0$ and $\frac{d}{dt}\left[e^T\eta\right] = \dot{e}^T\eta$, the derivative of V is

$$\begin{array}{l}\dot{V} = \dot{q}^T M\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}\dot{q} + \dot{e}^T K_p e + \eta^T \dot{q} \\ +\dot{e}^T\eta + tr\left[\tilde{W}^T K_w^{-1}\left(\frac{d}{dt}\tilde{W}\right)\right] \\ +\Lambda\dot{z}K_i^{-1}z + \dot{e}^T z + e^T\dot{z} - \alpha\left(\dot{e}^T M\dot{q} + e^T\dot{M}\dot{q} + e^T M\ddot{q}\right) \\ +\Lambda e^T K_d\dot{e}\end{array} \tag{10}$$

Proof of of PID+NN

Using Skew, the first three terms become

$$-\dot{q}^T\eta - \dot{q}^T K_d \dot{q} + \dot{q}^T z + \dot{q}^T \eta + \dot{q}^T \tilde{W}\sigma \tag{11}$$

And

$$\begin{aligned} \dot{V} \leq & -\left[\lambda_m\left(K_d\right) - \alpha\lambda_M\left(M\right) - \alpha k_c \left\|e\right\|\right]\left\|\dot{q}\right\|^2 \\ & -\left[\alpha\lambda_m\left(K_p\right) - \lambda_M\left(K_i\right) - \alpha k_g\right]\left\|e\right\|^2 \end{aligned} \tag{12}$$

If

$$\|e\| \leq \frac{\lambda_M(M)}{\alpha k_c} \tag{13}$$

and

$$\begin{array}{l} \lambda_m(K_d) \geq (1+\alpha)\,\lambda_M(M) \\ \lambda_m(K_p) \geq \frac{1}{\alpha}\lambda_M(K_i) + k_g \end{array} \tag{14}$$

Proof of of PID+NN

then $\dot{V} \leq 0$, $\|e\|$ decreases. Then (14) is established. Using (8) and $\lambda_m\left(K_i^{-1}\right) = \frac{1}{\lambda_M(K_i)}$, (14) is (2).

$\dot{V}$ is negative semi-definite. Define a ball Σ of radius $\sigma > 0$ centered at the origin of the state space, which satisfies these condition

$$\Sigma = \left\{ \tilde{q} : \|\tilde{q}\| \leq \frac{\lambda_M\left(M\right)}{\alpha k_c} = \sigma \right\} \tag{15}$$

$\dot{V}$ is negative semi-definite on the ball Σ.

There exists a ball Σ of radius $\sigma > 0$ centered at the origin of the state space on which $\dot{V} \leq 0$.

Proof of of PID+NN

The origin of the closed-loop equation (**??**) is a stable equilibrium. Since the closed-loop equation is autonomous, we use La Salle's theorem. Define Ω as

$$\begin{aligned}\Omega &= \left\{x\left(t\right) = \left[\tilde{q}, \dot{q}, \tilde{\xi}\right] \in R^{3n} : \dot{V} = 0\right\} \\ &= \left\{\tilde{\xi} \in R^{n} : \tilde{q} = 0 \in R^{n}, \dot{q} = 0 \in R^{n}\right\}\end{aligned} \tag{16}$$

From (10), $\dot{V} = 0$ if and only if $\tilde{q} = \dot{q} = 0$. For a solution $x\left(t\right)$ to belong to Ω for all $t \geq 0$, it is necessary and sufficient that $\tilde{q} = \dot{q} = 0$ for all $t \geq 0$.

Proof of of PID+NN

Therefore it must also hold that $\ddot{q} = 0$ for all $t \geq 0$. We conclude that from the closed-loop system (**??**), if $x(t) \in \Omega$ for all $t \geq 0$, then

$$\begin{aligned} \phi\left(q, \dot{q}\right) &= \phi\left(q^d, 0\right) = \tilde{\xi} + \phi\left(q^d, 0\right) \\ \dot{\tilde{\xi}} &= 0 \end{aligned} \tag{17}$$

implies that $\tilde{\xi} = 0$ for all $t \geq 0$. So $x(t) = \left[\tilde{q}, \dot{q}, \tilde{\xi}\right] = 0 \in R^{3n}$ is the only initial condition in Ω for which $x(t) \in \Omega$ for all $t \geq 0$.

Finally, we conclude from all this that the origin of the closed-loop system (**??**) is locally asymptotically stable. Because $\frac{1}{\alpha} \leq \lambda_m \left(K_i^{-1}\right) \lambda_m \left(K_p\right)$, the upper bound for $\|\tilde{q}\|$ can be

$$\|e\| \leq \frac{\lambda_M (M)}{k_c} \lambda_M (K_i) \lambda_m (K_p) \tag{18}$$

It establishes the semiglobal stability of our controller, in the sense that the domain of attraction can be arbitrarily enlarged with a suitable choice of the gains. Namely, increasing K_p the basin of attraction will grow.

Closed-loop tuning of PID control

Since the robot dynamic is not stable in open loop, it is impossible to send step commands to all joints of the robot to tune PID gains., we use

$$PD_1 = K_p e + K_d \dot{e}$$
$$PID_2 = K_p e + K_d \dot{e} + K_i \int_0^t e(\tau)\, d\tau$$

The closed-loop system with PD_1 is stable. If we apply PID_2 to the closed-loop system

$$M(q)\ddot{q} + C\left(q, \dot{q}\right)\dot{q} + \tilde{g}(q) + f\left(\dot{q}\right) - PD_1 = PID_2$$

The total control torque to the robot is

$$\tau = PID_2 + PD_1$$

if we tune PID controllers m times,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + f\left(\dot{q}\right) = \sum_{j=1}^{m} PID_j$$

Closed-loop tuning od PID control

PID gains are linear independent,
The refinement of PID is the same as adding a new PID controller. For PID_3

Closed-loop tuning od PID control

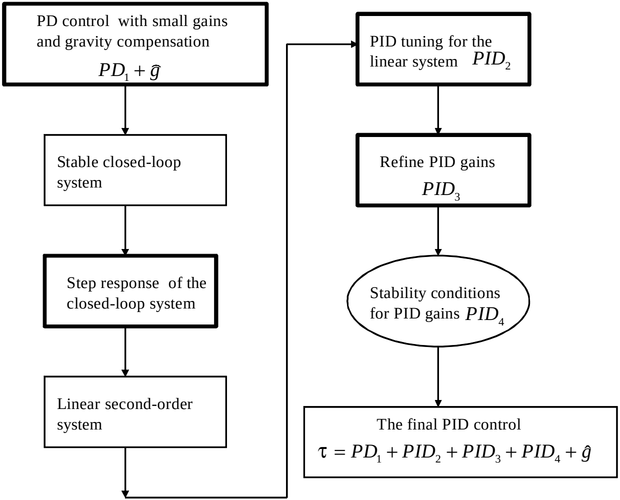


Figure: PID tuning scheme.

PID with velocity observer

PID

$$\begin{aligned} u &= K_p e + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e} \\ &= K_p \left(q^d - q\right) + K_i \int_0^t e(\tau)\, d\tau + K_d \left(\dot{q}^d - \dot{q}\right) \end{aligned}$$

The first-order filter

$$v(s) = \frac{bs}{s+a} q(s)$$

where $v(s)$ is an estimation of $\dot{q}$

The transfer function can be realized by

$$\begin{cases} \dot{x} = -A(x + Bq) \\ v = x + Bq \end{cases}$$

PD with velocity observer

The PID control becomes

$$\begin{aligned} u &= K_p e + K_d \upsilon + z + W\sigma(q) \\ \dot{z} &= K_i e \\ \dot{x} &= -A\left(x + Bq\right) \\ \upsilon &= x + Bq \end{aligned}$$

$$\|e\| \leq \frac{\lambda_m\left(M\right)}{\alpha k_c}\left[\lambda_m\left(B - \alpha I\right) - \frac{1}{2}\lambda_m\left(A\right)\right] + \frac{1}{\alpha}\|\upsilon\|$$

PD with velocity observer

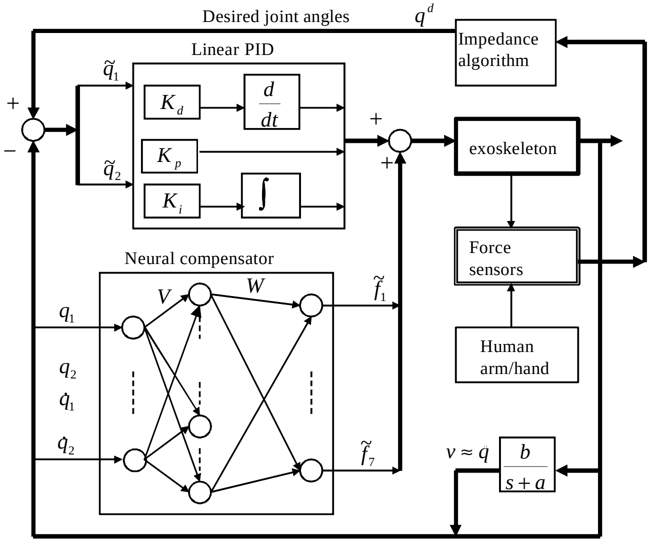


Figure:

PD control with fuzzy compensation

PD with neural network compensation

$$u = PD + NN$$

where

$$\hat{f} = g\left(q\right) + f\left(\dot{q}\right) = W_t \sigma(V_t x) + \varepsilon$$

l fuzzy rules

$$\text{R}^i\text{: IF } \left(q_1 \text{ is } A_{1i}^1\right) \text{ and } \left(q_2 \text{ is } A_{2i}^1\right) \text{ and } \cdots \left(q_n \text{ is } A_{ni}^1\right) \text{ THEN } \hat{f} \text{ is } B_{1i}$$

$$\hat{f} = \hat{W}_t \Phi\left(s\right)$$

$$\phi_i^p = \prod_{j=1}^{n} \mu_{A_{ji}^p} / \sum_{i=1}^{l} \prod_{j=1}^{n} \mu_{A_{ji}^p},$$

Training

$$\frac{d}{dt}\hat{W}_p = -K_w Z_p r_1^T$$

$$\frac{d}{dt}c_{ji}^p = -2k_c z_i^p \frac{\hat{w}_{pi}-\widehat{y}_p}{b_p} \frac{s_j - c_{ji}^p}{\left[\sigma_{ji}^p\right]^2} r_{1,p}^T$$

$$\frac{d}{dt}\sigma_{ji}^p = -2k_b z_i^p \frac{\hat{w}_{pi}-\widehat{y}_p}{b_p} \frac{\left(s_j - c_{ji}^p\right)^2}{\left[\sigma_{ji}^p\right]^3} r_{1,p}^T$$

PD control with fuzzy compensation and high-gain observer

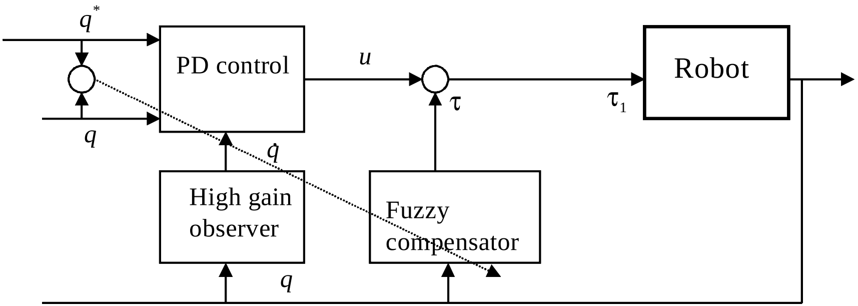


Figure: PD control with fuzzy compensation

PD control with sliding mode control

PD+SM

$$\tau = K_p e + K_d \dot{e} + K_s sgn(e)$$

The uncertainty is $f + G + F$. We have to select a very big sliding mode gain, such that

$$K_s > \bar{d}$$

PD control with sliding mode control and NN

$$u = K_p e + K_d \dot{e} + W\sigma(x) + K_s sgn(e)$$

$$G + F + f = W^* \sigma(x) + \eta$$

$$K_s > \bar{\eta} \ << \bar{d}$$

PD control with sliding mode control and NN

Theorem

If the updating law for the weights of neural networks is

$$\frac{d}{dt}W = K_w \sigma(x) r_t^T \tag{19}$$

the sliding mode gain

$$K_s > \bar{\eta}$$

$\bar{\eta}$ *is the upper bound of the neural approximation error*

$$f + G + F = W\sigma(x) + \bar{\eta}\ , \quad \|\eta\|^2 \leq \bar{\eta}$$

then the neural sliding mode PD control forces the tracking error asymptotic stability,

$$\lim_{t \to \infty} r_t = 0 \tag{20}$$

PD control with sliding mode control and NN

SERIAL Neural sliding mode PID control (PID+NN+SMC)

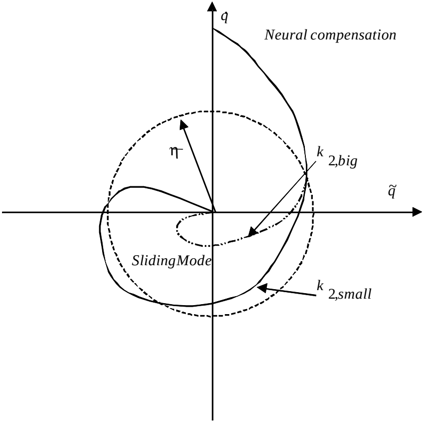


Figure: Tracking error with NN and SMC

PID control with sliding mode control and NN

$$u = K_p e + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e} + W\sigma(x) + K_s sgn(r)$$

$$u = u_1 + (1 - s_t)\, u_2$$

$$s_t = \begin{cases} 1 & \text{if } \|r\|_Q^2 \geq \bar{\eta} \\ 0 & \text{if } \|r\|_Q^2 < \bar{\eta} \end{cases}$$

PID control with sliding mode control and NN

Theorem

If the sliding mode satisfies

$$K_s > \bar{\eta}$$

where $\bar{\eta}$ *is the upper bound of the neural modeling error, then the neural sliding mode PD/PID control with the neural training law*

$$\frac{d}{dt}W = s_t K_w \sigma(x) r^T$$

make the tracking error r *is stable, and it converge to zero in finite time,*

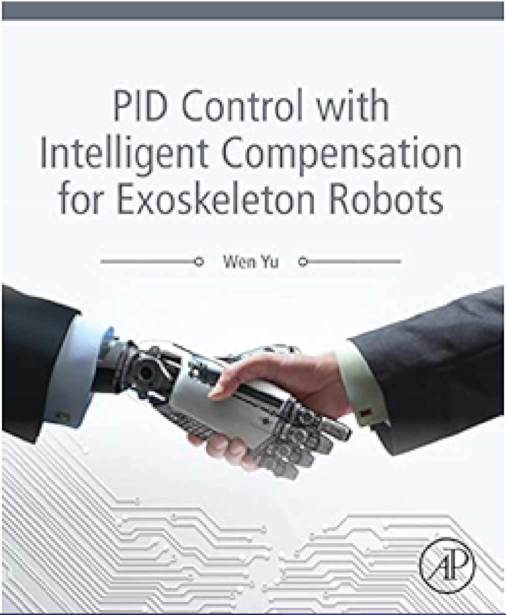
$$\|r\| \longrightarrow 0$$

Conclusion

Stability of PD and PID

1. $\longrightarrow$ Tuning of PID
2. $\longrightarrow$ PID+NN
3. $\longrightarrow$ PID+Fuzzy
4. PD$\longrightarrow$Velocity observer
5. PID+NN $\longrightarrow$PD+NN+SM

2018


PID Control with
Intelligent Compensation
for Exoskeleton Robots
Wen Yu