Home Exercises for the course

"Identification of Parameters, Filtering, Prediction and Smoothing of Dynamic Models"

Exercise 1 *Let us consider the following ARMA model*

$$\left.\begin{array}{c} x_{n+1} = a x_n + b u_n + \zeta_n, \\ \zeta_n = \xi_n + d_1 \xi_{n-1} + d_2 \xi_{n-2}, \\ x_n, u_n, \zeta_n \in R^1, \ n = 0, 1, ..., \end{array}\right\} \tag{1}$$

with

$$\left.\begin{array}{c} a = 0.5, \ b = 1, \ d_1 = 0.3, \ d_2 = -0.33, \\ x_0 = 5, \ u_n = \sin(0.3n), \end{array}\right\} \tag{2}$$

and $\{\xi_n\}_{n=0,1...}$ *is a stationary sequence of* ***independent Gaussian*** *random variables satisfying*

$$\left.\begin{array}{c} E\{\xi_n\} = 0, \ E\left\{\xi_n^2\right\} = \sigma^2 = 1, \\ E\{\xi_n \xi_k\} \overset{n \neq k}{=} 0, \ E\{\xi_n x_n\} = 0, \\ E\{\xi_n u_n\} = E\left\{\xi_{n-1} u_n\right\} = E\left\{\xi_{n-2} u_n\right\} = 0. \end{array}\right\} \tag{3}$$

The model (1) can be represented in the generalized regression format as

$$\boxed{\left.\begin{array}{c} x_{n+1} = c^{\intercal} z_n + \zeta_n, \\ c := \begin{pmatrix} a \\ b \end{pmatrix}, \ z_n := \begin{pmatrix} x_n \\ u_n \end{pmatrix} - \textit{generalized regression vector.} \end{array}\right\}} \tag{4}$$

Problem: estimate the vector c ***using in each time*** n ***the observations*** (x_{n+1}, z_n).

To do that let us apply the Instrumental Variable (IV) estimation algorithm

$$\boxed{\left.\begin{array}{c} c_{n+1} = c_n + \Gamma_n v_n \left(x_{n+1} - z_n^{\intercal} c_n\right), \ c_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\ \Gamma_n = \Gamma_{n-1} - \dfrac{\Gamma_{n-1} v_n z_n^{\intercal} \Gamma_{n-1}}{1 + z_n^{\intercal} \Gamma_{n-1} v_n}, \ z_n^{\intercal} \Gamma_{n-1} v_n \neq -1, \\ \Gamma_0 = \rho^{-1} I_{2\times 2}, \ \rho = 10^{-5}. \end{array}\right\}} \tag{5}$$

Notice that

- *for* $v_n = z_n$ *this is* ***Least Squares Method*** *(LSM);*
- *for* $v_n = z_{n-2}$ *this is* ***Instrumental Variables Method*** *(IVM);*

Show (by numerical simulations) that LSM method does not work in this example, but IVM correctly estimates unknown parameter c, namely,

$$c_n \underset{n\to\infty}{\to} c = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}.$$

Exercise 2