Lecture 13: Average Sub-Gradient Method in DNN Control (Continuation)

Plan of presentation

- Guidance Control of Underwater Autonomous Vehicle
- 2-4 steps of Backstepping
- Desired dynamics
- DNN version

2-nd step of Backstepping control (1)

- Considere now the situation when the desired control $\boldsymbol{v}^* = \begin{bmatrix} u^* & v^* & w^* \end{bmatrix}^\mathsf{T}$ at the first stage is fixed and equal to $\mathbf{u}_1^*$, given in the previouse theorem.
- Take as the second intermediate pseudo-control $\mathbf{u}_2 = \begin{bmatrix} \tau_u & q & r \end{bmatrix}^\mathsf{T}$. Then the equation for $\boldsymbol{v}^*$ can be represented as

$$\left.\begin{array}{c} \dot{\upsilon} = \mathbf{f}_2\left(\boldsymbol{v}^*\right) + \mathbf{B}_2\mathbf{u}_2 + \zeta_\upsilon, \\ \mathbf{f}_2\left(\boldsymbol{v}^*\right) := \begin{bmatrix} -I_1^{-1} d_1 u^* \\ -I_2^{-1} d_2 v^* \\ -I_3^{-1} d_3 w^* \end{bmatrix}, \\ \mathbf{B}_2 := \begin{bmatrix} I_1^{-1} & -I_1^{-1} I_3 w^* & I_1^{-1} I_2 v^* \\ 0 & 0 & -I_2^{-1} I_1 u^* \\ 0 & I_3^{-1} I_1 u^* & 0 \end{bmatrix}. \end{array}\right\} \quad (1)$$

- Define then the translation velocity tracking error as $\boldsymbol{\varphi}_2 = \boldsymbol{v} - \boldsymbol{v}^*$. Following the same scheme of representation as at the first stage we are able to formulate the tracking trajectory problem at this stage an optimization, realized by an uncertain controlable dynamic plant:

$$\left.\boxed{\begin{array}{c} J_2(\boldsymbol{\varphi}_2) = \sum_{i=1}^{3} \left|\varphi_{2,i}\right| \underset{t\to\infty}{\longrightarrow} \min_{\mathbf{u}_2(\cdot) \in U_{2,adm}} \\ \text{subjected to (1).} \end{array}}\right\} \quad (2)$$

The ideal $\mathbf{u}_2^*$ solving the problem (2) denote by $\mathbf{u}_2^* = \begin{bmatrix} \tau_u^* & (\boldsymbol{\omega}^*)^\intercal \end{bmatrix}^\intercal$.

Theorem

Under the accepted assumptions the intermediate pseudo-control $\mathbf{u}_2^*$, *realizing the soltion of the problem (2), satisfies the following ODE's*

$$\left.\begin{array}{l} \frac{d}{dt}\left(\mathbf{B}_2\mathbf{u}_2\right)+\mathbf{g}_2=-k_2\mathrm{Sign}\left(\mathbf{s}_2\right),\ \mathbf{u}_2^*\left(0\right)=\mathbf{u}_{2,0}^*,\ k_2>\dot{\zeta}_v^+, \\ \mathrm{Sign}\left(\mathbf{s}_2\right):=\left[\ \mathrm{sign}\left(\mathbf{s}_{2,1}\right),\quad \mathrm{sign}\left(\mathbf{s}_{2,2}\right),\quad \mathrm{sign}\left(\mathbf{s}_{2,3}\right)\ \right]^{\intercal}, \end{array}\right\} \quad (3)$$

$$\begin{array}{c} \mathbf{g}_2:=-\ddot{v}^*+\frac{\dot{v}-\dot{v}^*}{t+\theta}-\frac{\boldsymbol{v}-\boldsymbol{v}^*+\boldsymbol{\alpha}_1}{\left(t+\theta\right)^2} \\ -\frac{1}{t+\theta}\Gamma_2+\frac{1}{t+\theta}\partial J_{\mathbf{2}}(\boldsymbol{\varphi}_2)+\dot{f}_2\left(\boldsymbol{v}^*\right), \end{array} \quad (4)$$

2-nd step of Backstepping control (4)

Theorem (continuation)

where the integral sliding variable $\mathbf{s}_1$ *is defined as*

$$\left.\begin{array}{c}\mathbf{s}_2=\dot{\varphi}_2+\dfrac{\boldsymbol{\varphi}_2+\boldsymbol{\alpha}_2}{t+\theta}+\Gamma_2,\ \Gamma_2=\dfrac{1}{t+\theta}\displaystyle\int_{\tau=0}^{t}\partial J_{\mathbf{1}}(\boldsymbol{\varphi}_1)d\tau,\ t\geq 0,\ \theta>0,\\ \\ \partial J_{\mathbf{2}}(\boldsymbol{\varphi}_2)=\left[\ \mathrm{sign}\left(\varphi_{2,1}\right),\ \ \mathrm{sign}\left(\varphi_{2,2}\right),\ \ \mathrm{sign}\left(\varphi_{2,3}\right)\ \right]^{\intercal},\\ \boldsymbol{\alpha}_2=-\theta\dot{\varphi}_2\left(0\right)-\boldsymbol{\varphi}_2\left(0\right).\end{array}\right\}$$

It guarantees that

$$J_2(\boldsymbol{\varphi}_2\left(t\right))\leq\frac{\Phi_2}{t+\theta}\underset{t\to\infty}{\to}0,\ \Phi_2=\theta J_{\mathbf{2}}(\boldsymbol{\varphi}_2\left(0\right))+\frac{1}{2}\left\|\boldsymbol{\alpha}_2\right\|^2. \quad (5)$$

2-nd step of Backstepping control: proof

Proof.

[Proof of Theorem 1]

$$\dot{V}(\mathbf{s}_2) = \mathbf{s}_2^\intercal \dot{s}_2 =$$
$$\mathbf{s}_2^\intercal \left[\ddot{v} - \ddot{v}^* + \frac{\dot{v} - \dot{v}^*}{t+\theta} - \frac{\boldsymbol{v} - \boldsymbol{v}^* + \boldsymbol{\alpha}_2}{(t+\theta)^2} - \frac{1}{t+\theta}\Gamma_2 + \frac{1}{t+\theta}\partial J_{\mathbf{2}}(\boldsymbol{\varphi}_2) \right]$$
$$= \mathbf{s}_2^\intercal \left[\frac{d}{dt} (\mathbf{B}_2 \mathbf{u}_2) + \dot{\zeta}_v + \mathbf{g}_2 \right] = \mathbf{s}_2^\intercal \left[-k_2 \mathrm{Sign}(\mathbf{s}_2) + \dot{\zeta}_v \right],$$

and then the proof exactly folows the proof of Theorem about the first step. □

3-rd step of Backstepping control (1)

- The orientation dynamics can be represented as

$$\left.\begin{array}{c}\dot{\omega} = \mathbf{f}_3 + \mathbf{B}_3\mathbf{u}_3 + \zeta_{\omega},\\ \mathbf{f}_3 := f_{\omega}\left(\boldsymbol{v}, \boldsymbol{\omega}, \boldsymbol{\eta}\right) = \\ \left[\begin{array}{c} I_5^{-1}\left(I_3 - I_1\right)u^* w^* - \dfrac{d_5}{m}q^* - I_5^{-1} mghs_{\theta^*} \\ I_6^{-1}\left(I_1 - I_2\right)u^* v^* - I_6^{-1} d_6 r^* \end{array}\right] \\ \text{- is a vector measurable (available) on-line,} \\ \mathbf{u}_3 := \left(\begin{array}{c}\tau_q \\ \tau_r\end{array}\right), \ \mathbf{B}_3 := \left[\begin{array}{cc}\dfrac{1}{I_5} & 0 \\ 0 & \dfrac{1}{I_6}\end{array}\right].\end{array}\right\} \quad (6)$$

- In the third stage, consider that the dynamic of the angular velocity is regulated by the pseudocontrol action defined by truster τ_q and τ_r.

Define the angular velocity tracking error as $\boldsymbol{\varphi}_3 = \boldsymbol{\omega} - \boldsymbol{\omega}^*$, where $\boldsymbol{\omega}^*$ is obtained from the previous stage. The corresponding tracking trajectory problem at this stage may be formulated as an optimization, realized by an uncertain controllable dynamic plant:

$$\left.\boxed{\begin{array}{c} J_3(\boldsymbol{\varphi}_3) = \sum\limits_{i=1}^{2} \left|\varphi_{3,i}\right| \underset{t\to\infty}{\rightarrow} \min\limits_{\mathbf{u}_3(\cdot)\,\in\, U_{3,adm}} \\ \text{subjected to (6).} \end{array}}\right\} \tag{7}$$

Denote by $\mathbf{u}_3^* = \begin{bmatrix} \tau_q^* & \tau_r^* \end{bmatrix}^{\mathsf{T}}$ the soltion of the optimization problem (7).

3-rd step of Backstepping control (3)

Theorem

Under the accepted assumptions the intermediate pseudo-control $\mathbf{u}_3^*$, *realizing the soltion of the problem (7), satisfies the following ODE's*

$$\left.\begin{array}{c} \frac{d}{dt}\left(\mathbf{B}_3\mathbf{u}_3\right)+\mathbf{g}_3=-k_3\mathrm{Sign}\left(\mathbf{s}_2\right),\ \mathbf{u}_3^*\left(0\right)=\mathbf{u}_{3,0}^*,\ k_3>\dot{\zeta}_{\omega}^{+}, \\ \mathrm{Sign}\left(\mathbf{s}_3\right):=\left[\ \mathrm{sign}\left(\mathbf{s}_{3,1}\right),\quad \mathrm{sign}\left(\mathbf{s}_{3,2}\right)\ \right]^{\intercal}, \end{array}\right\} \tag{8}$$

$$\begin{array}{c} \mathbf{g}_3:=-\ddot{\omega}^*+\frac{\dot{\omega}-\dot{\omega}^*}{t+\theta}-\frac{\boldsymbol{\omega}-\boldsymbol{\omega}^*+\boldsymbol{\alpha}_3}{\left(t+\theta\right)^2} \\ -\frac{1}{t+\theta}\Gamma_3+\frac{1}{t+\theta}\partial J_{\mathbf{3}}(\boldsymbol{\varphi}_3)+\dot{f}_3\left(\boldsymbol{\omega}^*\right), \end{array} \tag{9}$$

3-rd step of Backstepping control (3)

Theorem (continuation)

where the integral sliding variable $\mathbf{s}_3$ *is defined as*

$$\left.\begin{array}{c} \mathbf{s}_3 = \dot{\varphi}_3 + \dfrac{\boldsymbol{\varphi}_3 + \boldsymbol{\alpha}_3}{t+\theta} + \Gamma_3, \\ \Gamma_3 = \dfrac{1}{t+\theta} \displaystyle\int_{\tau=0}^{t} \partial J_{\mathbf{3}}(\boldsymbol{\varphi}_3) d\tau, \ t \geq 0, \ \theta > 0, \\ \partial J_{\mathbf{3}}(\boldsymbol{\varphi}_3) = \left[\ \operatorname{sign}(\varphi_{3,1}), \quad \operatorname{sign}(\varphi_{3,2}), \quad \operatorname{sign}(\varphi_{3,3}) \ \right]^{\intercal} \\ \boldsymbol{\alpha}_3 = -\theta\dot{\varphi}_3(0) - \boldsymbol{\varphi}_3(0). \end{array}\right\} \qquad (10)$$

It guarantees that

$$J_3(\boldsymbol{\varphi}_3(t)) \leq \frac{\Phi_3}{t+\theta} \underset{t\to\infty}{\to} 0, \ \Phi_3 = \theta J_{\mathbf{3}}(\boldsymbol{\varphi}_3(0)) + \frac{1}{2}\|\boldsymbol{\alpha}_3\|^2. \qquad (11)$$

Proof.

[Proof of Theorem 3]

$$\dot{V}(\mathbf{s}_3) = \mathbf{s}_3^{\intercal}\dot{s}_3 =$$
$$\mathbf{s}_3^{\intercal}\left[\ddot{\omega} - \ddot{\omega}^* + \frac{\dot{\omega} - \dot{\omega}^*}{t+\theta} - \frac{\boldsymbol{\omega} - \boldsymbol{\omega}^* + \boldsymbol{\alpha}_3}{(t+\theta)^2} - \frac{1}{t+\theta}\Gamma_3 + \frac{1}{t+\theta}\partial J_{\mathbf{3}}(\boldsymbol{\varphi}_3)\right]$$
$$= \mathbf{s}_2^{\intercal}\left[\frac{d}{dt}(\mathbf{B}_3\mathbf{u}_3) + \mathbf{g}_3 + \dot{\zeta}_\omega\right] = \mathbf{s}_2^{\intercal}\left[-k_3\mathrm{Sign}(\mathbf{s}_3) + \dot{\zeta}_\omega\right],$$

and then the proof folows the proof of Theorem concerning the 1-st step. □

4-th step of Backstepping control (1): torque tracking

- The dynamic actuators model is

$$\boxed{\dot{\tau} = Z_E \mathbf{g} + \mathbf{B}_4 \mathbf{u}_4, \mathbf{B}_4 := Z_E} \tag{12}$$

- Here

$$\mathbf{u}_4 = \begin{bmatrix} \boldsymbol{\nu}_u & \boldsymbol{\nu}_q & \boldsymbol{\nu}_r \end{bmatrix}^{\mathsf{T}}$$

is the last intermediate control affecting the general dynamics.

- Define the last tracking error as $\varphi_4 = \tau - \tau^*$, where τ^* are obtained from the previous stages.
- Then the corresponding tracking problem at the last stage may be formulated as an optimization, realized by an uncertain controllable dynamic plant:

$$\left.\begin{array}{c} J_4(\boldsymbol{\varphi}_4) = \sum_{i=1}^{3} \left| \varphi_{4,i} \right| \underset{t \to \infty}{\rightarrow} \underset{\mathbf{u}_4(\cdot) \in U_{4,adm}}{\min} \\ \text{subjected to (12).} \end{array}\right\} \tag{13}$$

Denote by $u_4^* = \begin{bmatrix} \boldsymbol{\nu}_u^* & \boldsymbol{\nu}_q^* & \boldsymbol{\nu}_r^* \end{bmatrix}^{\mathsf{T}}$ the soltion of the problem (13).

4-th step of Backstepping control (2): torque tracking

Theorem

Under the accepted assumptions the intermediate pseudo-control $\mathbf{u}_4^*$*, realizing the soltion of the problem (13), satisfies the following ODE's*

$$\left.\begin{array}{c} \frac{d}{dt}\left(\mathbf{B}_4\mathbf{u}_4\right) + \mathbf{g}_4 = -k_4 \mathrm{Sign}\left(\mathbf{s}_4\right), \; \mathbf{u}_4^*\left(0\right) = \mathbf{u}_{4,0}^*, \; k_4 > \left\|Z_E\right\| \dot{g}^+, \\ \mathrm{Sign}\left(\mathbf{s}_4\right) := \left[\begin{array}{ccc} \mathrm{sign}\left(\mathbf{s}_{4,1}\right), & \mathrm{sign}\left(\mathbf{s}_{4,2}\right) & \mathrm{sign}\left(\mathbf{s}_{4,3}\right) \end{array}\right]^{\intercal}, \\ \mathbf{g}_4 := -\ddot{\tau}^* + \frac{\dot{\tau} - \dot{\tau}^*}{t+\theta} - \frac{\boldsymbol{\tau} - \boldsymbol{\tau}^* + \boldsymbol{\alpha}_4}{\left(t+\theta\right)^2} \\ -\frac{1}{t+\theta}\Gamma_4 + \frac{1}{t+\theta}\partial J_{\mathbf{4}}(\boldsymbol{\varphi}_4), \end{array}\right\} \tag{14}$$

Theorem (continuation)

where the integral sliding variable $\mathbf{s}_3$ *is defined as*

$$\left.\begin{array}{c} \mathbf{s}_4 = \dot{\varphi}_4 + \dfrac{\boldsymbol{\varphi}_4 + \boldsymbol{\alpha}_4}{t+\theta} + \Gamma_4, \\ \Gamma_4 = \dfrac{1}{t+\theta} \displaystyle\int\limits_{\tau=0}^{t} \partial J_{\mathbf{4}}(\boldsymbol{\varphi}_4) d\tau, \ t \geq 0, \ \kappa > 0, \\ \partial J_{\mathbf{4}}(\boldsymbol{\varphi}_4) = \left[\ \mathrm{sign}\left(\varphi_{4,1}\right), \quad \mathrm{sign}\left(\varphi_{4,2}\right), \quad \mathrm{sign}\left(\varphi_{4,3}\right) \ \right]^{\intercal}, \\ \boldsymbol{\alpha}_4 = -\theta \dot{\varphi}_4\left(0\right) - \boldsymbol{\varphi}_4\left(0\right). \end{array}\right\} \quad (15)$$

It guarantees that

$$J_4(\boldsymbol{\varphi}_4(t)) \leq \frac{\Phi_4}{t+\theta} \underset{t\to\infty}{\to} 0, \ \Phi_4 = \theta J_{\mathbf{4}}(\boldsymbol{\varphi}_4(0)) + \frac{1}{2}\left\|\boldsymbol{\alpha}_4\right\|^2. \quad (16)$$

Proof.

[Proof of Theorem 5]

$$\left.\begin{array}{c}\dot{V}(\mathbf{s}_4) = \mathbf{s}_4^\intercal \dot{\mathbf{s}}_4 = \\ \mathbf{s}_4^\intercal \left[\ddot{\tau} - \ddot{\tau}^* + \frac{\dot{\tau}-\dot{\tau}^*}{t+\theta} - \frac{\boldsymbol{\tau}-\boldsymbol{\tau}^*+\boldsymbol{\alpha}_4}{(t+\theta)^2} - \frac{1}{t+\theta}\Gamma_4 + \frac{1}{t+\theta}\partial J_{\mathbf{4}}(\boldsymbol{\varphi}_4)\right] \\ = \mathbf{s}_4^\intercal \left[\frac{d}{dt}(\mathbf{B}_4\mathbf{u}_4) + \mathbf{g}_4 + Z_E \mathbf{g}\right] \leq \\ -k_4 \mathbf{s}_4^\intercal \mathrm{Sign}(\mathbf{s}_4) + \|\mathbf{s}_4\| \|Z_E\| \dot{g}^+ \leq \\ -\|\mathbf{s}_4\| \left(k_4 - \|Z_E\| \dot{g}^+\right),\end{array}\right\}$$

and then the proof folows the proof of Theorem from the previouse lecture. □

Diagram of control structure

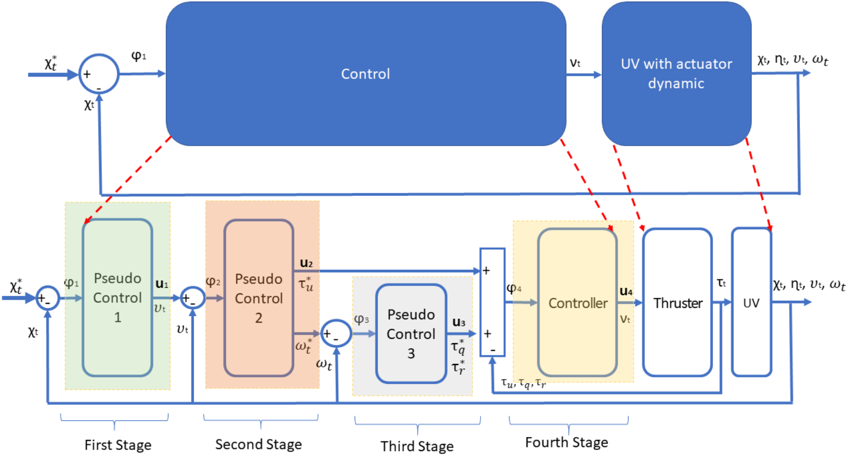

Figure 1: Diagram of control structure

Recall the model of UVAV: $v = \begin{bmatrix} \mathbf{v}_1^\intercal & \mathbf{v}_2^\intercal \end{bmatrix}^\intercal \in R^5$ where $v_1 = \begin{bmatrix} u & v & w \end{bmatrix}^\intercal$ the translation velocity and $v_2 = \begin{bmatrix} q & r \end{bmatrix}^\intercal$ is angular velocity. The dynamic model of the state vectors of the UAV is given by

$$\left.\begin{aligned} \frac{d}{dt}\mathbf{x}_t &= \Theta\left(\mathbf{x}_t\right)\mathbf{v}_t + \mathbf{f}_1\left(\mathbf{x}_t, \mathbf{v}_t\right), \\ \frac{d}{dt}\mathbf{v}_t &= \mathbf{f}_\mathbf{v}\left(\mathbf{x}_t, \mathbf{v}_t\right) + \mathbf{B}_\mathbf{v}\boldsymbol{\tau}_t + \mathbf{f}_2\left(\mathbf{x}_t, \mathbf{v}_t, \boldsymbol{\tau}_t\right). \end{aligned}\right\} \tag{17}$$

Define the state $\hat{\zeta}_t = \begin{bmatrix} \hat{\mathbf{x}}_t^\intercal & \hat{\mathbf{v}}_t^\intercal \end{bmatrix}^\intercal \in \mathbb{R}^{10}$ of DNNO:

$$\left.\frac{d}{dt}\hat{\zeta}_t = \Sigma\left(\zeta_t, \hat{\zeta}_t, \mathbf{y}_t^* \mid \hat{W}_t\right), \ \frac{d}{dt}\hat{W}_t = \Omega\left(\zeta_t, \hat{\zeta}_t\right).\right\} \tag{18}$$

Here $\hat{\zeta}_t$ is treated as the current estimation of the vector $\zeta_t = \begin{bmatrix} \mathbf{x}_t^\intercal & \mathbf{v}_t^\intercal \end{bmatrix}^\intercal$ with the structure dynamics Σ containg the adaptive parameters $\hat{W}_t$ which are adjusted on-line to provide a "good quality" of the identification process. Only the state $\mathbf{x}_t$ is observable on-line.

DNNO model: open format

Define the dynamics of DNNO as

$$\left.\begin{array}{c}\frac{d}{dt}\hat{\zeta}_t = A\hat{\zeta}_t + F\left(\hat{\zeta}_t\right) + \hat{W}_{1,t}\sigma_1\left(\hat{\zeta}_t\right) + \\ \hat{W}_{2,t}\sigma_2\left(\hat{\zeta}_t\right)\hat{\mathbf{v}}_t + \hat{W}_{3,t}\sigma_3\left(\hat{\zeta}_t\right)\mathbf{u}_t + L\left(\mathbf{x}_{1,t} - \mathbf{x}^*_{1,t}\right), \\ \hat{\zeta}_t = \left[\begin{array}{cc}\hat{\mathbf{x}}_t^\intercal & \hat{\mathbf{v}}_t^\intercal\end{array}\right]^\intercal, \; F\left(\hat{\zeta}_t\right) = \left[\begin{array}{c}\Theta\left(\hat{\mathbf{x}}_t\right)\hat{\mathbf{v}}_t \\ 0\end{array}\right].\end{array}\right\} \quad (19)$$

Here $\hat{\zeta}_t \in \mathbb{R}^{10}$ is the state of DNNO, while $\hat{W}_{1,t} \in \mathbb{R}^{10\times p_{21}}$, $\hat{W}_{2,t} \in \mathbb{R}^{10\times p_{11}}$ and $\hat{W}_{3,t} \in \mathbb{R}^{10\times p_{22}}$ are the time varying matrix weights parameters. The vector $\hat{\mathbf{v}}_t \in \mathbb{R}^5$ is the estimate of $\mathbf{v}_t$ and

$$\left.\begin{array}{c}\mathbf{x}_t = C_0\zeta_t, C_0 = \left[\begin{array}{cc}I_{5x5} & 0_{5x5}\end{array}\right], \\ \\ \mathbf{x}_{1,t} = C\mathbf{x}_t = (x_t, y_t, z_t)^\intercal = CC_0\zeta_t \in \mathbb{R}^3.\end{array}\right\} \quad (20)$$

The error of tracking:

$$\boxed{\delta_t = \mathbf{x}_{1,t} - \mathbf{x}^*_{1,t} = CC_0\left(\zeta_t - \zeta^*_t\right)} \tag{21}$$

Notice that

$$\boxed{\begin{array}{c} \left\|\delta_t\right\| = \left\|CC_0\left(\zeta_t - \zeta^*_t\right)\right\| = \left\|CC_0\left(\left[\zeta_t - \hat{\zeta}_t\right] + \left[\hat{\zeta}_t - \zeta^*_t\right]\right)\right\| \\ \leq \left\|CC_0\left(\zeta_t - \hat{\zeta}_t\right)\right\| + \left\|CC_0\left(\hat{\zeta}_t - \zeta^*_t\right)\right\|, \end{array}}$$

To characterize the quality of the tracking process $\hat{\delta}_t = \hat{x}_{1,t} - x^*_{1,t} \in R^3$ let us use the loss function

$$\boxed{J\left(\hat{\delta}_t\right) := \sum_{i=1}^{3} \left|\hat{x}_{1i,t} - x^*_{1i,t}\right|.} \tag{22}$$

DNNO model: sliding variable

Introduce also for all $t \geq t_0$ the auxilary vector function $\mathbf{s}_t \in R^3$ which below is referred to as "*sliding variable*":

$$\left.\begin{aligned} \mathbf{s}_t &= \frac{d}{dt}\hat{\delta}_t + \frac{\hat{\delta}_t + \boldsymbol{\eta}}{t+\theta} + \tilde{\mathbf{G}}_t, \ \boldsymbol{\eta} \in \mathbb{R}^3, \\ \tilde{\mathbf{G}}_t &:= \frac{1}{t+\theta} \int_{\tau=t_0}^{t} \mathbf{a}\left(\hat{\delta}_t\right) d\tau, \ \theta > 0, \end{aligned}\right\} \tag{23}$$

where $\mathbf{a}\left(\hat{\delta}_t\right) = \partial J\left(\hat{\delta}_t\right)$ is the subgradient of the function $J\left(\hat{\delta}_t\right)$ (22) in the point $\hat{\delta}_t$. Suppose that

$$\mathbf{s}_t = 0 \text{ for all } t \geq t_0. \tag{24}$$

Then the following result holds.

Lemma

If (24) holds, then we may guarantee the functional convergence

$$\boxed{J\left(\hat{\delta}_t\right) \leq \frac{\Phi_{t_0}}{t+\theta} \underset{t\to\infty}{\to} 0,\ \Phi_{t_0} = (t_0+\theta)\,F\left(\hat{\delta}_{t_0}\right) + \frac{1}{2}\left\|\boldsymbol{\eta}\right\|^2.} \tag{25}$$

Remark

To have $\mathbf{s}_{t_0} = 0$ *at time* $t_0 = 0$, *namely,*

$$\boxed{\mathbf{s}_{t_0} = \frac{d}{dt}\hat{\delta}_{t_0=0} + \frac{\hat{\delta}_{t_0=0}+\boldsymbol{\eta}}{\theta} + \tilde{\mathbf{G}}_{t_0=0} = \frac{d}{dt}\hat{\delta}_0 + \frac{\hat{\delta}_0+\boldsymbol{\eta}}{\theta} = 0}$$

it is sufficient to take

$$\boxed{\boldsymbol{\eta} = -\theta\frac{d}{dt}\hat{\delta}_0 - \hat{\delta}_0,} \tag{26}$$

.

DNNO model: dynamics of sliding variable

Let us represent the DNNI dynamics (19) the short following format:

$$\left.\begin{array}{c}\frac{d}{dt}\hat{\zeta}_t = \mathbf{g}_t + \mathcal{B}_t \mathbf{u}_t,\ \hat{\zeta}_t = \left[\begin{array}{cc}\hat{\mathbf{x}}_t^{\intercal} & \hat{\mathbf{v}}_t^{\intercal}\end{array}\right]^{\intercal}, \\ \\ \mathbf{g}_t := A\hat{\zeta}_t + F\left(\hat{\zeta}_t\right) + \hat{W}_{1,t}\sigma_1\left(\hat{\zeta}_t\right) \\ +\hat{W}_{2,t}\sigma_2\left(\hat{\zeta}_t\right)\mathbf{v}_t + L\left(\mathbf{x}_{1,t} - \mathbf{x}_{1,t}^*\right), \\ \\ \mathcal{B}_t := \hat{W}_{3,t}\sigma_3\left(\hat{\zeta}_t\right),\ F\left(\hat{\zeta}_t\right) = \left[\begin{array}{c}\Theta\left(\hat{\mathbf{x}}_t\right)\hat{\mathbf{v}}_t \\ 0\end{array}\right].\end{array}\right\} \quad (27)$$

DNNO model: dynamics of sliding variable

In view of (23), we have

$$
\begin{aligned}
\dot{s}_t = {} & \frac{d^2}{dt^2}\left[CC_0\left(\hat{\zeta}_t - \zeta_t^*\right)\right] + \frac{1}{t+\theta}\frac{d}{dt}\left[CC_0\left(\hat{\zeta}_t - \zeta_t^*\right)\right] \\
& -\frac{\hat{\delta}_t + \eta}{(t+\theta)^2} - \frac{1}{(t+\theta)^2}\int_{\tau=t_0}^{t} \mathbf{a}\left(\hat{\delta}_\tau\right) d\tau + \frac{\mathbf{a}\left(\hat{\delta}_t\right)}{t+\theta} = \\
& CC_0\frac{d}{dt}\left[\frac{d}{dt}\hat{\zeta}_t - \dot{\zeta}_t^*\right] + \frac{CC_0}{t+\theta}\left[\frac{d}{dt}\hat{\zeta}_t - \zeta_t^*\right] + \\
& \frac{1}{t+\theta}\left(\mathbf{a}\left(\hat{\delta}_t\right) - \left[\frac{\hat{\delta}_t + \eta}{t+\theta} + \tilde{\mathbf{G}}_t\right]\right) = \\
& CC_0\frac{d}{dt}\left[\mathbf{g}_t + \mathcal{B}_t\mathbf{u}_t\right] + \frac{CC_0}{t+\theta}\left[\mathbf{g}_t + \mathcal{B}_t\mathbf{u}_t\right] + \\
& CC_0\ddot{\zeta}_t^* + \frac{1}{t+\theta}\left(\mathbf{a}\left(\hat{\delta}_t\right) - \left[\frac{\hat{\delta}_t + \eta}{t+\theta} + \tilde{\mathbf{G}}_t\right] - CC_0\dot{\zeta}_t^*\right) = \\
& CC_0\left[\mathcal{B}_t\dot{u}_t + \left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta}\mathcal{B}_t\right)\mathbf{u}_t\right] + \mathbf{p}_t\ ,
\end{aligned}
$$

DNNO model: dynamics of sliding variable

So, we have

$$\boxed{\dot{s}_t = CC_0 \left[\mathcal{B}_t \dot{u}_t + \left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta} \mathcal{B}_t \right) \mathbf{u}_t \right] + \mathbf{p}_t \,,} \tag{28}$$

where

$$\boxed{\left. \begin{array}{c} \mathbf{p}_t := CC_0 \left(\ddot{\zeta}_t^* - \dfrac{1}{t+\theta} \dot{\zeta}_t^* + \dot{g}_t + \dfrac{1}{t+\theta} \mathbf{g}_t \right) \\ + \dfrac{1}{t+\theta} \left(\mathbf{a} \left(\hat{\delta}_t \right) - \left[\dfrac{\hat{\delta}_t + \eta}{t+\theta} + \tilde{\mathbf{G}}_t \right] \right) . \end{array} \right\}} \tag{29}$$

Let the control action $\mathbf{u}_t$ satisfies the following ODE:

$$\boxed{CC_0\left[\mathcal{B}_t\dot{u}_t + \left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta}\mathcal{B}_t\right)\mathbf{u}_t\right] + \mathbf{p}_t = -k\mathrm{Sign}\left(\mathbf{s}_t\right),\ k > 0,} \quad (30)$$

or in the resolving format

$$\boxed{\begin{array}{c} \dot{u}_t = \mathcal{B}_t^+\left[-\left(CC_0\right)^+\left[\mathbf{p}_t + k\mathrm{Sign}\left(\mathbf{s}_t\right)\right] - \left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta}\mathcal{B}_t\right)\mathbf{u}_t\right] \\ = -\mathcal{B}_t^+\left(CC_0\right)^+\left[\mathbf{p}_t + k\mathrm{Sign}\left(\mathbf{s}_t\right)\right] - \mathcal{B}_t^+\left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta}\mathcal{B}_t\right)\mathbf{u}_t, \\ \\ \mathcal{B}_t^+\left(CC_0\right)^+ = \left[\hat{W}_{3,t}\sigma_3\left(\hat{\zeta}_t\right)\right]^+\left[\begin{array}{cc} I_{3x3} & O_{3x2} \end{array}\right]\left[\begin{array}{cc} I_{5x5} & O_{5x5} \end{array}\right] \\ \bar{W}_3 = B_1 W_3\ \left(\bar{W}_3 \in \mathbb{R}^{10\times p_{22}}\right),\ B_1 = \left[\begin{array}{cc} 0_{5\times5} & I_{5\times5} \end{array}\right]^{\mathsf{T}} \end{array}}$$

DNNO model: Lyapunov function analysis

The Lyapunov function $V(s) = \frac{1}{2}\|\mathbf{s}\|^2$ we have

$$\boxed{\begin{aligned}\dot{V}(\mathbf{s}_t) &= \mathbf{s}_t^{\intercal}\dot{s}_t = \mathbf{s}_t^{\intercal}\left(CC_0\left[\mathcal{B}_t\dot{u}_t + \left(\dot{\mathcal{B}}_t + \frac{1}{t+\theta}\mathcal{B}_t\right)\mathbf{u}_t\right] + \mathbf{p}_t\right) \\ &= -k\mathbf{s}_t^{\intercal}\mathrm{Sign}(\mathbf{s}_t) = -k\sum_{i=1}^{3}|s_{i,t}| \leq -k\|\mathbf{s}_t\| = -k\sqrt{2}\sqrt{V(\mathbf{s}_t)},\end{aligned}}$$

implying

$$\boxed{0 \leq \|\mathbf{s}_t\| = \sqrt{V(\mathbf{s}_t)} \leq \sqrt{V(\mathbf{s}_0)} - \frac{k}{\sqrt{2}}t = \frac{1}{\sqrt{2}}(\|\mathbf{s}_0\| - kt)}$$

which means that for all $t \geq t_{reach} = \frac{\|\mathbf{s}_0\|}{k}$ we obtain the desired regime

$$\boxed{\mathbf{s}_t = 0 \text{ for all } t \geq t_{reach}.}$$

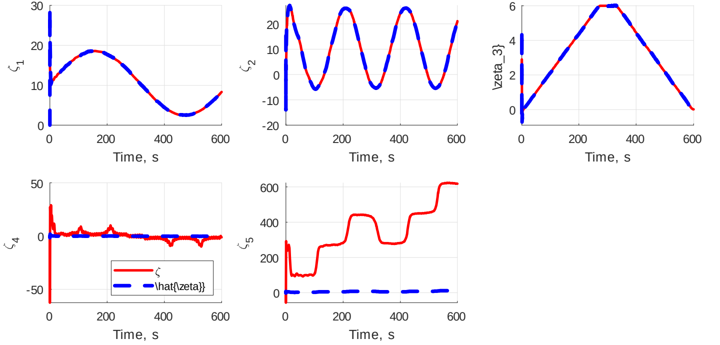

Figure 2: The position and velocity of the underwater vehicle model and by the neural network model

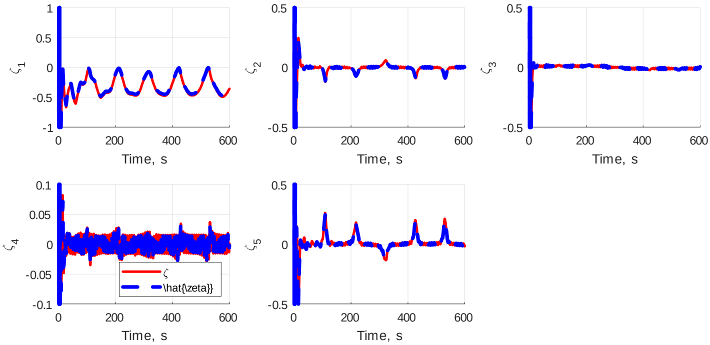

Figure 3: Translaton velocity and angular velosity state by the underwater vehicle and the neural network model

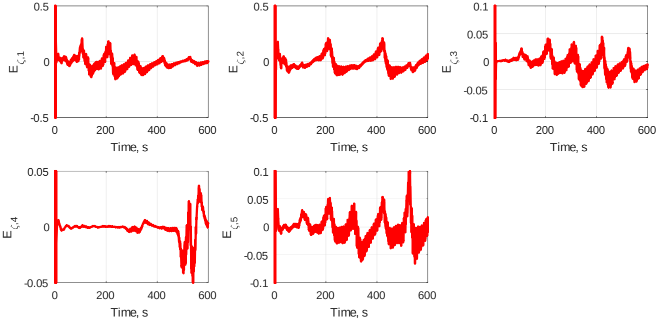

Figure 4: Error estimation by the Neural Network model in position and orientation

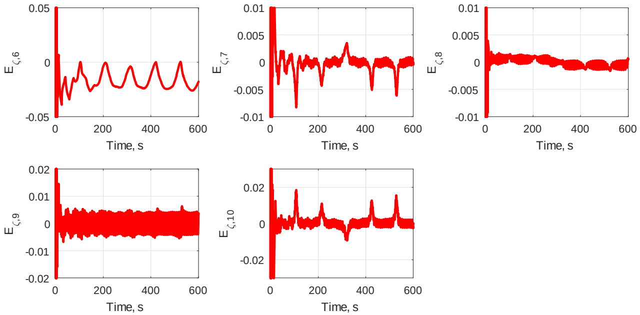

Figure 5: Estimation error by the Neural Network model in translation and angular velocity

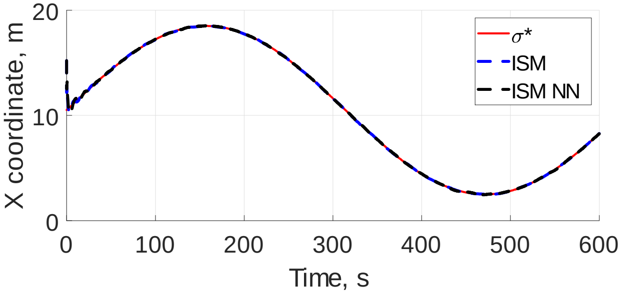

Figure 6: Tracking trajectory in x axis by ISM and ISM-NN controllers

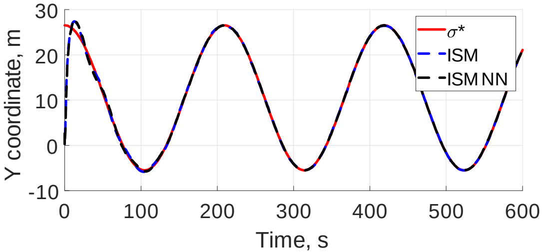

Figure 7: Tracking trajectory in Y axis by ISM and ISM-NN controllers

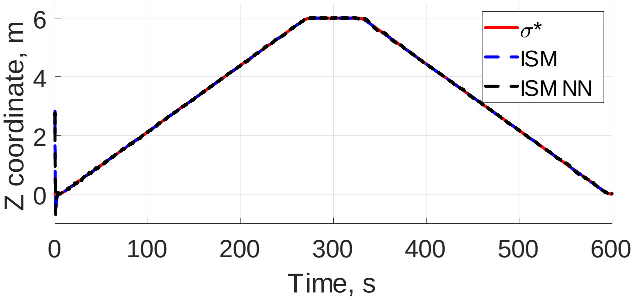

Figure 8: Tracking trajectory in z axis by PD, ISM and ISM-O controllers

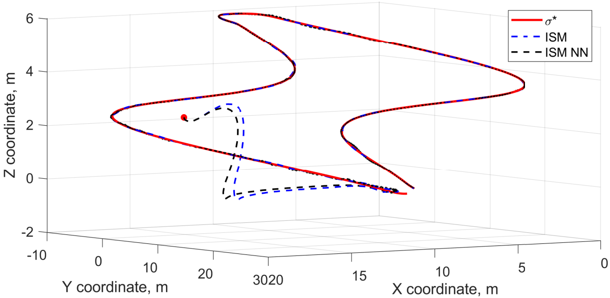

Figure 9: Tracking trajectory in 3D space by ISM and ISM-NN controllers

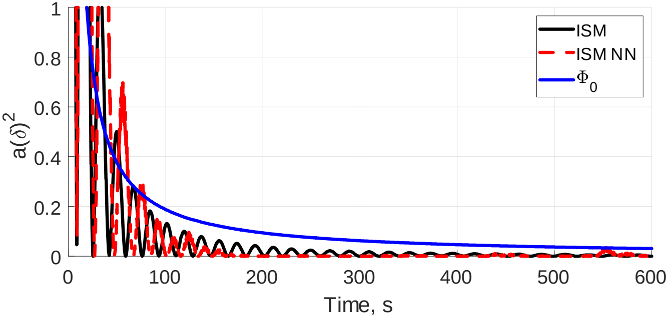

Figure 10: Cost function of the tracking error δ

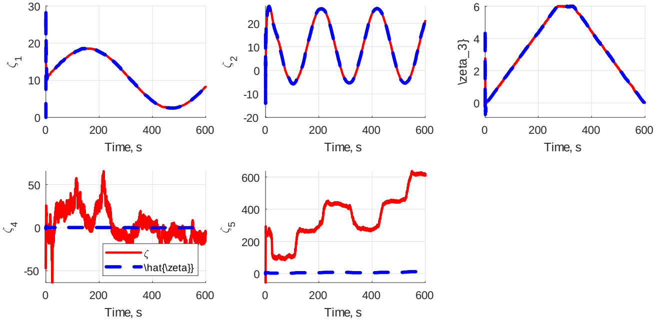

Figure 11: The position and velocity of the underwater vehicle model and by the neural network model

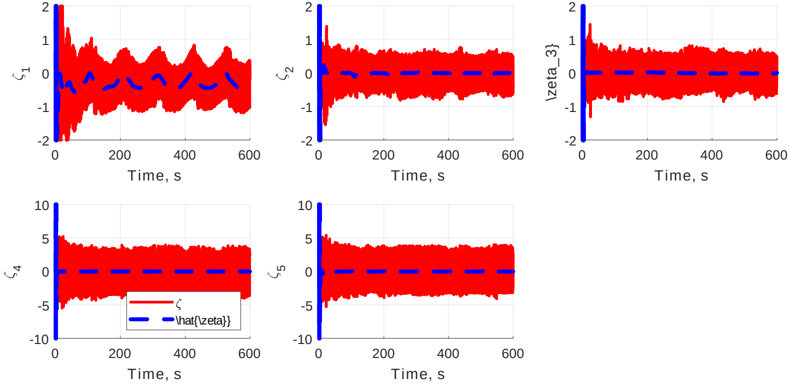

Figure 12: Translaton velocity and angular velosity state by the underwater vehicle and the neural network model

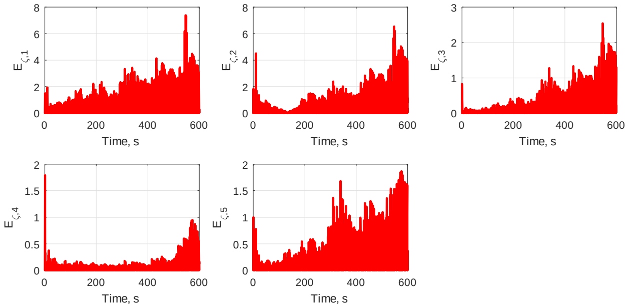

Figure 13: Error estimation by the Neural Network model in position and orientation

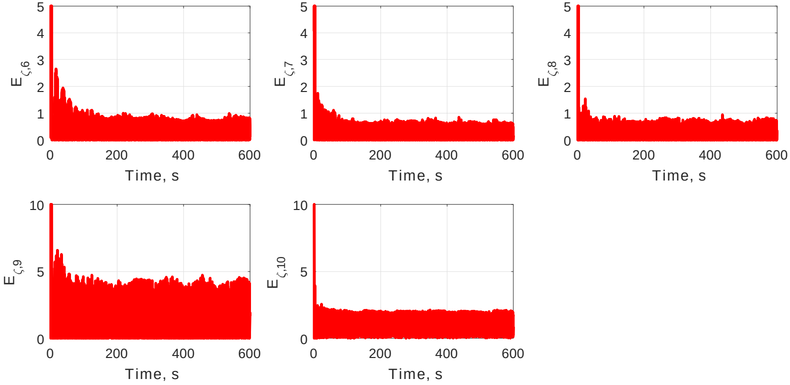

Figure 14: Estimation error by the Neural Network model in translation and angular velocity

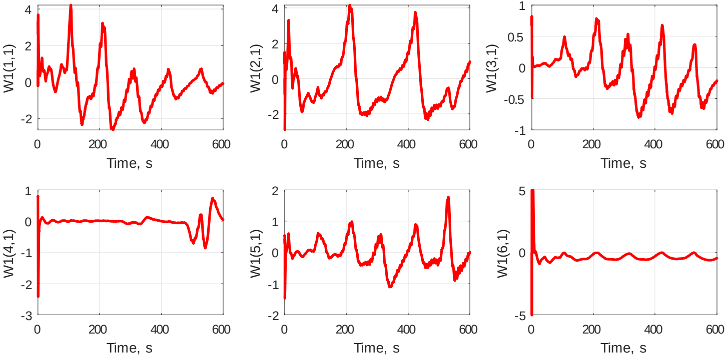

Figure 15: Weights of matrix W_1

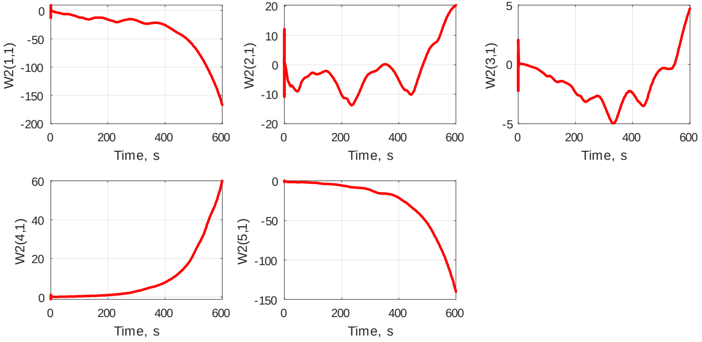

Figure 16: Weights of matrix W_2

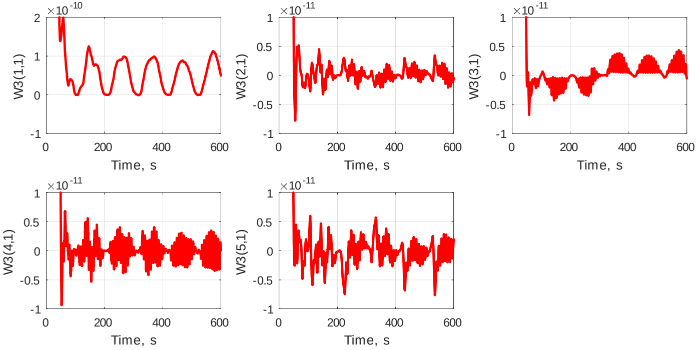

Figure 17: Weights of matrix W_1